kg^{-1} (Table 2 vs. Fig. 2–7). As the STP approached and exceeded 100 mg P kg^{-1}, the soil P solubilities approached the OCP isotherm line. In addition, ortho-P concentrations in the 0.01 *M* $CaCl_2$ extract generally exceeded 1 mg L^{-1} when the STP was >100 mg P kg^{-1}, both more than sufficient for optimum plant growth on calcareous soils and at concentrations that suggest that ortho-P would move deeper into the soil profile with leaching water.

REFERENCES

Berg, R.D., and D.L. Carter. 1980. Furrow erosion and sediment losses on irrigated cropland. J. Soil Water Conserv. 35:267–270.

Carter, D.L., R.D. Berg, and B.J. Sanders. 1985. The effect of furrow irrigation erosion on crop productivity. Soil Sci. Soc. Am. J. 49:207–2.

Cary, J.W., and W.W. Rasmussen. 1979. Response of three irrigated crops to deep tillage of a semiarid silt loam. Soil Sci. Soc. Am. J. 43:574–577.

Fox, R.L., and E.J. Kamprath. 1970. Phosphate isotherms for evaluating the phosphorus requirements of soils. Soil Sci. Soc. Am. Proc. 34:902–906.

Griffin, R.A., and J.J. Jurinak. 1973. Estimation of activity coefficients from the electrical conductivity of natural aquatic systems and soil extracts. Soil Sci. 116:26–30.

Jones, S.B., C.L. Hansen, and C.W. Robbins. 1993. Chemical oxygen demand fate from cottage cheese (acid) whey applied to a sodic soil. Arid Soil Res. Rehab. 7:71–79.

Kitson, R.E., and M.G. Mellon. 1944. Colorimetric determination of phosphorus as Molybdivanado phosphoric acid. Ind. Eng. Chem., Anal Ed. 16:379–383.

Lindsay, W.L. 1979. Chemical equilibria in soils. Wiley International, New York.

Nishimoto, R.K., R.L. Fox, and P.E. Parvin. 1977. Response of vegetable crops to phosphorus concentrations in soil solution. J. Am. Soc. Hort. Sci. 102:705–709.

Robbins, C.W. 1977. Hydraulic conductivity and moisture retention characteristics of southern Idaho's silt loam soils. Res. Bull. 99. Idaho Agricultural Experiment Stn., Moscow.

Robbins, C.W. 1989. Sample preparation for determining ions in dark colored sodic soil extracts. Soil Sci Soc. Am. J. 53:721–725.

Robbins, C.W., B.E. Mackey, and L.L. Freeborn. 1997. Fertilizer source and crop rotation effects on exposed subsoil dry bean production. Soil Sci. Soc. Am. J. 61:1221–1225.

Robbins, C.W., R.J. Wagenet, and J.J. Jurinak. 1980. A combined salt transport-chemical equilibrium model for calcareous and gypsiferous soils. Soil Sci. Soc. Am. J. 44:1191–1194.

Robbins, C.W., and C.L. Wiegand. 1990. Field and laboratory measurements. p. 201–219. *In* K.K. Tanji (ed.) Agricultural salinity assessment and management. Am. Soc. Civ. Eng., New York.

SAS Institute. 1989. SAS/STAT user's guide. Version 6. 4th ed. Vol. 2. SAS Inst., Cary, NC.

Stumm, W., and J.J. Morgan. 1970. Aquatic chemistry. John Wiley & Sons, New York.

University of Idaho. 1998. Idaho fertilizer guides. College of Agriculture, Moscow, ID

Watanabe, F.S., and S.R. Olsen. 1965. Test of an ascorbic acid method for determining phosphorus in water and $NaHCO_3$ extracts from soils. Soil Sci. Soc. Am. Proc. 29:677–678.

Westermann, D.T. 1992. Lime effects on phosphorus availability in a calcareous soil. Soil Sci. Soc. Am. J. 56:489–494.

Submeter Spatial Variability of Selected Soil and Bermudagrass Production Variables

J. B. Solie,* W. R. Raun, and M. L. Stone

ABSTRACT

The optimal resolution at which soil and plant variables should be sensed and treated is not well defined. This study was conducted to determine the semivariance range where soil test and plant variables were related, and to estimate the minimum spatial scale at which variable rate applications of nutrients should be made. Soil and plant analyses were performed in 490 0.3- by 0.3-m plots from bermudagrass (*Cynodon dactylon* L.) sod at two locations. Eight soil cores (0–15 cm deep) were collected and composited from each 0.3- by 0.3-m plot. Semivariance analysis was used to estimate the range over which samples of the five soil variables (total N, extractable P, and K, organic C, and pH) and two plant variables (forage total N and biomass) were related. Semivariance statistics including the nugget, sill, correlation range, and integral scale were calculated. Correlation ranges were between 1.9 and 11.4 m with corresponding integral scales between 0.5 and 2.1 m. At one location, P exhibited nested sills with multiple ranges. Results indicate that the fundamental field-element dimensions (the area over which variable rate fertilizer applicators should sense and apply materials) is likely to be 1.0 by 1.0 m or smaller. To describe the variability encountered in these experiments, soil and plant measurements should be made at the meter or submeter level.

J.B. Solie and M.L. Stone, Dep. of Biosystems and Agricultural Engineering, Oklahoma State Univ., Stillwater, OK 74078; W.R. Raun, Dep. of Plant and Soil Sciences, Oklahoma State Univ., Stillwater, OK 74078. Received 29 Jan. 1998. *Corresponding author (jsolie@okstate.edu).

Published in Soil Sci. Soc. Am. J. 63:1724–1733 (1999).

A PROTOTYPE variable-rate applicator has been developed (Stone et al., 1996) that optically senses and corrects N deficiency occurring in areas less than 1 by 1 m in wheat (*Triticum aestivum* L.) and bermudagrass. Sensing and variable-rate application of other nutrients at the submeter level will become practical as real-time sensors are developed. Solie et al. (1996) demonstrated that the correlation range between optically sensed plant N fell between 0.70 and 4.46 m with 1.4 m being common to all transects measured. They proposed that an area existed of approximately 1.5 by 1.5 m that provided the most precise measure of the actual nutrition needs of the crop, and that real-time, variable-rate sensor applicators should be designed to sense and treat areas at that scale.

Terminology for the area to be sensed with a single measurement and treated at a single rate has not been standardized. Terms used by researchers and practitioners include picture element or pixel, cell, element, sensed area or treated area, grid-cell, and management zone. Solie et al. (1996) proposed that the sensed and treated area be called the *field-element* and defined the area that provided the most precise measure of the nutrient level available to the plant as the *fundamental field-element*.

Table 1. Recently reported semivariance ranges and coefficients of variation for soil and plant variables sampled at intervals of 30 m or greater, or sampled randomly.

	Coefficient of Variation					Range				
Variable	Boyer et al. (1991)	Cahn et al. (1994)	Nolin et al. (1996)	Gupta et al. (1997)	Miller et al. (1988)	Boyer et al. (1991)	Cahn et al. (1994)	Nolin et al. (1996)	Gupta et al. (1997)	Miller et al. (1988)
	%					m				
NO_3–N	–	59.7	35.7	16–40	–	–	5	198	5–28	–
P	99.4	36.2	35.6	24–50	–	37	≥54	39	7–60	–
K	–	42.8	31	13–25	–	–	30–50	17	40–157	–
Organic C	36.1	16.3	26.1	–	18	56	≥50	343	–	50
pH	7.6	–	5.6	–	5.8	53	–	165	–	–
Biomass	56.8	–		–	25.4	46	–	–	–	75

Components of the ideal field element size were reported by Raun et al. (1998), who also demonstrated that large spatial variability existed at the submeter level for both mobile and immobile soil nutrients. Bermudagrass forage yields ranged from 1300 to as much as 10 000 kg ha^{-1}, and soil pH ranged as much as 2 pH units in a 2- by 21-m area. Phosphorus and potassium fertilizer recommendations based on individual 0.3- by 0.3-m plots, ranged from 0 to 31 kg ha^{-1} and 0 to 107 kg ha^{-1}, respectively (Raun et al., 1998). If current and future sensors and controllers will be used to treat meter-scale areas, questions that must be answered include (i) what are the fundamental field-element sizes for plant and soil variables of interest? and (ii) what error is introduced in the value of the measured variable as the size of the sensed field-element increases above that fundamental size?

Reported research on horizontal soil variability has been conducted almost exclusively at sampling intervals much greater than 1 m, although significant differences in soil test parameters in the vertical direction have been detected at less than 0.1-m intervals (Follett and Peterson, 1988). Researchers studying horizontal variability used semivariance analysis to determine the range in which measurements of soil properties were related (Table 1). With one exception, these studies sampled at separation distances (i.e., distance separating consecutive measurements) much greater than 1 m. Chancellor and Goronea (1994) sampled at a separation distance of 1 m and reported total mineral N coefficients of variation between 34.5 to 66.0% and the semivariance range of 19.5 m. However, their analysis of errors when N content in a field-element was used to predict N levels of field-elements at increasing distances from the predictor element suggested that the semivariance range existed near 1 m. CVs of measured data reported in these papers ranged from 5 to 343% and generally were near 50%.

Reported semivariance ranges were much larger than those reported by Solie et al. (1996). Sensors that measure soil and plant nutrients should be designed to scan areas in which the measured variable has maximum relatedness, if nutrient applications are to be optimized. Two experiments were conducted that measured seven soil and plant variables with 0.30- by 0.30-m resolution. Analyses were performed on the bermudagrass plant and soil data presented by Raun et al. (1998) to meet the following objectives: (i) determine the range and integral scales within which measurements of these soil and plant variables were related, and (ii) evaluate the error in estimating the value of the fundamental field-elements of the variables when sensing over larger areas.

SEMIVARIANCE ANALYSIS

Basic semivariance analysis theory and procedures to define relatedness between samples of spatially varying soil and plant variables have been outlined in numerous texts (Journel and Huijbregts, 1978; Royle et al., 1980; Hohn, 1988; Isaaks and Srivastava, 1989) and journal articles (Miller et al., 1988; Boyer et al., 1991; Cahn et al., 1994; Nolin et al., 1996; Gupta et al., 1997). These publications define semivariance, $\gamma(h)$, of all samples separated by a Vector h as:

$$\gamma(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} [z(x_i) - z(x_i + h)]^2 \qquad [1]$$

where $z(x_i)$ and $z(x_i + h)$ are experimental measures of any two points separated by the Vector h, and $N(h)$ is the number of experimental pairs separated by h. However, certain aspects of semivariance analysis specifically related to this paper have received limited emphasis. These topics include pseudocycling and nested structures, estimation of regions with a high degree of relatedness, and regularization and deconvolution between point and finite dimensioned data.

A phenomenon recognized in mining geostatistics, but not frequently discussed with respect to soil nutrient distribution, is the existence of multiple sills nested within a semivariogram (Journel and Huijbregts, 1978; Hohn, 1988). These sills are associated with physical phenomena occurring at scales of different orders of magnitude. Sills can be nested within a semivariogram, with semivariogram ranges in the order of millimeters to kilometers. Detection of ranges nested within larger ranges requires sample spacing shorter than the minimum range to be detected. However, as a general practice, apparent nested sills should be considered real only when their ranges can be associated with identifiable physical phenomena (Isaaks and Srivastava, 1989).

Many semivariograms appear to oscillate about the sill. The oscillation, termed pseudocycling, is defined as the apparent periodic cycling or oscillation of the magnitude of the variable over distance and is a common phenomenon with minerals (Hohn, 1988). Normally, changes in magnitude are random or aperiodic even though they appear periodic. However, certain geological formations are periodic in nature and can be characterized by semivariograms. Periodic variation in soil variables can be induced by fertilizer applicators. Solie et al. (1987) measured dynamic deposition patterns of floatation-tire liquid applicators and showed that these machines produced CVs ranging from 15 to 55%. Deposition patterns were cyclical with a random component. Solie et al. (1994) also investigated the dynamic patterns of floatation-tire pneumatic granular fertilizer applicators. They showed that these machines produced CVs of 14 to 22%. Fourier analysis showed that variability was associated with, among other factors, outlet spacing, swath width, and a random component.

Correlation exists between samples within the semivariogram range (Journel and Huijbregts, 1978). Beyond the range, the semivariance becomes approximately equal to the population variance of the measured variable. Measurements with separation distances greater than or equal to the range are unrelated. A distance less than the range has been proposed that would be more appropriate to define the region containing closely related samples. This vector has been termed the integral scale (Russo and Jury, 1987; Matheron, 1989) or the mean correlation distance (Han et al., 1994). The integral scale was derived from the autocorrelation function:

$$\rho(h) = \frac{(c_0 + c_1) - \gamma(h)}{c_0 + c_1} \quad [2]$$

where c_0 is the nugget semivariance and $c_0 + c_1$ equals the covariance $C(h)$ at $h = 0$. The integral scale J is defined as:

$$J = \int_0^a \rho(h)\mathrm{d}h \quad [3]$$

where a is the range.

Absent a biological basis for determining the distance at which soil and plant variables are highly related, the integral scale provides an objective procedure to estimate that distance.

Russo and Jury (1987) concluded, on the basis of an analysis of 100 independent simulated realizations, that the number of sample pairs with separation distances less than the range and the orientation of those pairs affected the values of the variogram range and integral scale. Their analysis showed that transect sampling underestimated the magnitude of the integral scale by at least a factor of two. They concluded that only with a large number of sample pairs ($N > 100$) does the estimated correlation scale (derived from the fitted semivariogram) approach the value of the corresponding scale of the underlying process. Further, they concluded that the semivariance pairs should not be collected along transects, because this sampling strategy tended to limit the number of available sample pairs.

Semivariance analysis assumes that measurements are punctual, i.e., made at points located in three-dimensional space. In practice, all measurements are made over a finite volume or area. Data collected from soil cores are routinely treated as point data when separation distances are sufficiently large. To estimate mean levels of a variable in a volume of soil, point data are integrated over that volume. This volume is generally referred to as the semivariance support. For purposes of this discussion, the term support will be used to refer to the volume, area, or line over which a measurement is made when performing geostatistical analysis. This is not to be confused with the term field-element which refers to the soil surface area over which measurements are made and fertilizers or pesticides applied by a machine. One or more supports can constitute a field-element. The geometry of the support may be simplified to two dimensions (area) or one (line) depending on the relative magnitudes of the dimensions of the support volume. The relationship between point and finite support measurement is defined as follows (Journel and Huijbregts, 1978):

$$z(x) = \frac{1}{v}\int_{v(x)} z(y)\mathrm{d}y \quad [4]$$

The mean value $z(x)$ is defined as the regularization of the Point Variable $z(y)$ for the Volume $v(x)$. In the geostatistical literature, inference of the point semivariance from a regularized variogram is referred to as *deregularization* or *deconvolution*, a term used in other disciplines such as digital signal processing for the same procedure.

The relationship between the point semivariance and the regularized semivariance is:

$$\gamma_v(h) = \bar{\gamma}(v, v') - \bar{\gamma}(v, v) \quad [5]$$

Where:

- $\gamma_v(h)$ is the semivariance at separation distance h regularized over the support (volume area or line) v.
- $\bar{\gamma}(v,v')$ is the mean value of all possible values of $\gamma(h)$ when one extremity of the vector h lies within the domain v and the other extremity lies within the domain v'.
- $\bar{\gamma}(v,v)$ is the mean value of the semivariance values within a volume support.

$\bar{\gamma}(v, v')$ is calculated by:

$$\bar{\gamma}(v,v') = \frac{1}{vv'}\int_v \mathrm{d}x \int_{v'} \gamma(x - x')\mathrm{d}x \quad [6]$$

The domain v can be one, two, or three-dimensional and in fact is represented by double integrals over dx and dy in two dimensions and triple integrals over du, dx, and dy in three dimensions.

Journel and Huijbregts (1978) provide analytical and graphical solutions to Eq. [5] for regularization of point data for two basic support geometries. They also described procedures to estimate the deregularized or point semivariogram.

Regularization can be important when the support dimensions approach the support scale v. At separation distances $A,v << h$, the difference between the regularized semivariance and the point semivariance can be considered constant and equal to $\bar{\gamma}(v,v')$. This difference varies appreciably as $h \rightarrow A$, v (Journel and Huijbregts, 1978). The relationship between the regularized range and regularized sill to the point variogram range and sill are functions of the point variogram and the geometry of the support.

Sensors such as those described by Stone et al. (1996) measure rectangular or square field-elements. The geometric relationships among the separation distances of two regularized rectangular supports v and v' and points at their geometric centroids are defined in Fig. 1. Equation [6] can be solved for a specific point-semivariance transition model to calculate the effect of support geometry on the semivariogram. Consider two rectangular supports of dimension l_x and l_y whose centroids are separated by a distance h. Each support is subdivided into elements of dimensions dx and dy and dx = dy. An element in support v is located at coordinates x_1 and y_1 and an element in support v' is located at x_2 and y_2. The magnitude of a Vector $\vec{r}$ linking these elements is:

$$|\vec{r}| = [(x_2 - x_1)^2 + (y_2 - y_1)^2]^{0.5} \quad [7]$$

Equation [6] can be integrated numerically. If a rectangular support has m elements along the x-axis and n elements along the y-axis then the numerical solution to Eq. [6] is:

$$\bar{\gamma}(v, v') = \frac{1}{nmnm}\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{n}\sum_{l=1}^{m} \gamma(x_{i,j} - x'_{k,l}) \quad [8]$$

For a unit spherical model:

$$\bar{\gamma}(x_{i,j} - x'_{k,l}) = 1.5\frac{r}{a} - 0.5\frac{r^3}{a^3} \qquad \forall r = |h| \in [0, a] \quad [9]$$

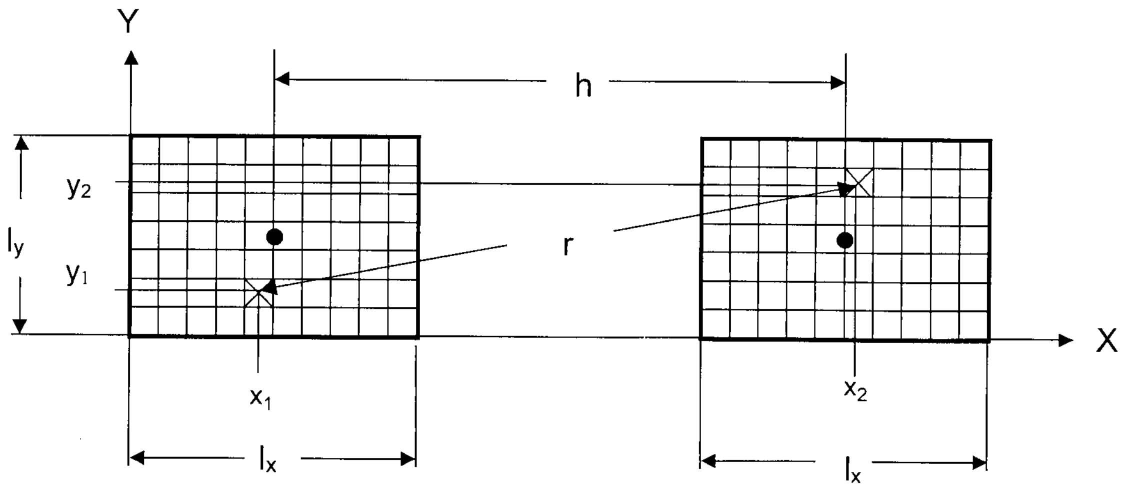

Fig. 1. Dimension diagram used for determining the effect on the semivariogram range of regularization to rectangular supports v and v' with dimensions l_x and l_y separated by a lag distance h. The centroid of the support is located at coordinates ($l_x/2$, $l_y/2$). The magnitude of the Vector r is the distance separating any two measurements over areas of dimension dx and dy that are located at (x_1, y_1) and (x_2, y_2) within supports v and v', respectively.

$$a = \text{range}$$

and

$$1 = \text{Sill} \qquad \forall r = |h| \in [\geq a]$$

The Vector r is defined by Eq. [7].

Normally, the experimenter collecting data over a regularized support does not know the point transition model. However, Journel and Huijbregts (1978) stated that the minimum value r_{min} and maximum value r_{max} of the deregularized range fell within the supports v and v' where $h = a$, the regularized range (Fig. 1). The minimum possible value of the deregularized range can be estimated as:

$$|\overrightarrow{r_{min}}| = a - l_x \qquad [10]$$

Equation [10] is true for all points along the inside boundaries of supports v and v', where the coordinates of pairs of points in the y-dimension are the same. The maximum possible value of the deregularized range can be estimated as:

$$|\overrightarrow{r_{min}}| = [(l_x + a)^2 + (l_y)^2]^{0.5} \qquad [11]$$

Equation [11] is true for opposite corners of the outside boundaries of supports v and v'. The minimum and maximum values of a reregularized range with a different dimensioned support can be estimated by applying Eq. [10] to the minimum value and Eq. [11] to the maximum value of the deregularized range. Journel and Huijbregts (1978) recommended that the estimated deregularized point range be reregularized to a different sized support, and those values be compared to the experimentally determined range to determine that the point estimates of the range were valid.

MATERIALS AND METHODS

Five soil variables, total soil N, extractable P and K, organic C, and pH and two plant variables, forage N and biomass, were measured in two experiments. The experiments were conducted on two established bermudagrass sods located at the Efaw Experiment Station farm, Stillwater, OK, and at Burneyville, OK. The soil at Efaw was a Norge loam (fine-silty, mixed thermic Udic Paleustoll) and at Burneyville, a Minco fine sandy loam (coarse-silty, mixed thermic Udic Haplustoll). The site at Efaw was mowed each year but was not pastured or harvested for hay. At Burneyville, the site was part of a large pasture (200 ha) that was annually grazed. No fertilizer had been applied at either location in at least 5 yr and both sites have undergone limited tillage in the last 10 yr. During extensive periods over the last 50 yr, both sites were tilled on a frequent basis since disk formed terraces remain. At each location, a visually homogeneous 2.13- by 21.3-m area was selected for intensive soil sampling. Each area was subdivided into 490 0.3- by 0.3-m supports. Eight soil cores (13 mm in diameter and 0–150 mm deep) were collected and composited from each 0.3- by 0.3- support. Compositing the samples effectively regularized the data to a 0.30- by 0.30-m area. The 0.30-m dimension corresponded to the horizontal root radius of the plant in the top 30 cm of the soil with no competition among plants (Lersten, 1987). Raun et al. (1998) presented a complete description of the soil test procedures, and analyses of sampling and laboratory error. Data were subjected to a standard descriptive statistical analysis.

Semivariance Analysis

Semivariance analysis was used to estimate the range over which samples of the five soil variables and two plant variables were related. Semivariograms were calculated by Eq. [1] along the major (21.3 m) axis of the experiments. The major axis semivariogram was important, because it was anticipated that, when sensors were incorporated into a variable rate applicator, they would be mounted parallel to the boom. Isaaks and Srivastava (1989) recommended that semivariograms be calculated unidirectionally. Semivariograms were calculated for the 490 sample data sets and for each of the seven 70-sample contiguous transects constituting the data sets. The maximum separation distance was 18.29 m (covering 60 0.30- by 0.30-m supports) when semivariance analysis was performed unidirectionally over the entire data set. The maximum separation distance used to calculate semivariance along individual transects was 35 supports (10.67 m). To determine the effect on the semivariograms of measurement over a larger support, data were averaged over 2 by 2 support arrays and semivariance calculated from the averaged data. When semivariograms were erratic, data were examined and outliers deleted that affected the semivariogram using the procedure of Isaaks and Srivastava (1989). Outliers were deleted until the semivariogram displayed the expected form. Nine or fewer data points were deleted from the Burneyville data set. These outliers appeared to be associated with cattle dung, e.g., a single data point in the P data set whose value was 124 $\mu g\ g^{-1}$ caused most of the distortion. This procedure removed the effects of manure deposits. No data were deleted in the Efaw data set.

Semivariance statistics calculated included the nugget, sill, and range of relatedness. Semivariances were plotted as scatter diagrams and visually examined to locate sills. Data files were clipped where semivariance data departed from the sill. A linear-plateau (linear to a sill) function was fitted to the data (Solie et al., 1996) when the relationship between the semivariance and separation distance in the transition region was linear. The standard exponential and spherical semivariogram transition functions (Isaaks and Srivastava, 1989) were also fitted to the data by the curve fitting program TableCurve (SPSS, 1997). The range of the exponential curve was defined as the separation distance at which the semivariance was 99% of the semivariance at $h = \infty$. Journel and Huijbregts (1978) recommended that the range of the exponential models be set at 95% of the semivariance at $h = \infty$. The data sets used in this paper were rich and well behaved. Inspection of the semivariograms showed that fixing the range at 95% greatly underestimated the actual range. The 99% standard much more closely approached the true range. Criteria for selecting the transition model included (i) the highest coefficient of determination, (ii) visual evaluation of how well the model fit the semivariance values at the shortest separation distances, and (iii) how well the model fit semivariance values in the region about the range.

To evaluate the effects of regularization on semivariogram statistics, theoretical regularized semivariograms were calculated from a point spherical transition model and experimentally determined ranges deregularized and reregularized to a different dimensioned support. Equation [8] was integrated for a unit spherical model whose point range, 1.72 m, was near the minimum range for the 14 semivariograms. The finite point dimensions $dx = dy = 0.030$ m were close to the diameter of a standard soil probe. The spherical model nugget value was 0.302. Support dimensions of the generated semivariograms were 0.15 by 0.15 m, 0.30 by 0.30 m, 0.61 by 0.61 m, 0.91 by 0.91 m, and 1.22 by 1.22 m. To develop reregularized semivariograms from the experimental data, the results of Eq. [8] were applied to Eq. [5]. Semivariogram statistics were calculated by the previously outlined procedure.

Minimum and maximum values of the deregularized experimental semivariograms were calculated by Eq. [10] and [11] for the 0.30- by 0.30-m data. Those values were reregularized to a 0.61- by 0.61-m support. Experimental data were averaged over 0.61- by 0.61-m supports, semivariograms constructed, and ranges calculated. The 0.61- by 0.61-m ranges were compared to the reregularized ranges to determine if they fell between the minimum and maximum values.

Field-Element Size Effect on Sensing Error

Assuming that the fundamental field-element size can be deduced on either a biological or geostatistical basis, the effect on sensing and application error from using a larger than optimum sized field-element can be assessed. The dispersion variance is a measure of the change in the variance as the size of the sensed area changes (Issaks and Srivastava, 1989) and is directly linked to semivariance. The dispersion variance is calculated by:

$$\sigma^2(a,b) = \frac{1}{kl}\sum_{j=1}^{l}\sum_{i=1}^{k}(z_{v_i} - m_{v_j}) \qquad [8]$$

where $\sigma^2(a,b)$ is the dispersion variance of the sensed support a constituting a field-element b, k is the number of sensed supports constituting a field-element, l is the number of sensed field-elements constituting the area or field, z_{vi} is the value of the sensed variable for a single support, and m_{vj} is the mean value of the variable measured over a field-element.

The mean error between the measured field-element and fundamental field-element is a direct measure of the magnitude of the error arising from sensing a variable over a volume or area larger than optimum. This statistic is also a measure of the degree to which material applied to correct a deficiency will be over or under applied. Engineers customarily use this statistic to evaluate the effectiveness of their sensor–control systems. The mean error was defined as:

$$E_{Appl.} = \frac{1}{l}\sum_{j=1}^{l}\frac{1}{m_{v_j}}\sum_{i=1}^{k}(z_{v_i} - m_{v_j}) \qquad [9]$$

where $E_{appl.}$ is the error in sensing and potential error in application.

To calculate the dispersion variance and the mean error, each data set was subdivided into field-elements (Table 2) whose sizes were evenly divisible into the 6- by 66-support array and the two statistics calculated for each size field-element.

RESULTS AND DISCUSSION

Semivariance Analysis

Semivariograms of the data displayed unique responses (Fig. 2 and 3). All but two semivariograms, K at Burneyville (Fig. 2e) and pH at Efaw (Fig. 2j), were nonlinear between the nugget and sill. Semivariograms for P at both locations, soil organic C at Efaw, and biomass at Burneyville displayed possible nested sills (Fig. 2c, 2d, 2h, and 3c, respectively). Drift in the measured variable may have occurred in these semivariograms as well as in the total soil N semivariograms at both locations.

The semivariance curve for pH at Efaw (Fig. 2j) had a region between the nugget and peak semivariance that was divided into two distinct zones described by different curves. There was no defined sill linking the zones, but there was a sigmoidal type transition. Although a nested exponential model could be fitted to these data, a linear model produced the highest coefficient of determination in the first zone. We considered this semivariance curve to have two separate and distinct ranges. However, the semivariogram for a 0.61- by 0.61-m support exhibited only the first range. This is contrasted with the Efaw forage N semivariogram (Fig. 3b) where a single monotonic curve with a range of 11.35 m

Table 2. Field-element configurations used to calculate the error from the average value of the field-element compared with the measured values of each plot constituting that element.

Field-element size	Number of supports per field-element	Plot configuration	Number of field-elements
m		m	
1.83 by 20.12	396	6 by 66	1†
1.83 by 1.83	36	6 by 6	11
0.91 by 0.91	9	3 by 3	44*
0.61 by 0.61	4	2 by 2	99*

† Number of field-elements occupying the 6 by 66 plot area.

appeared to best fit the data, even with apparent inflections in the data.

The accuracy of the previous observations depends on whether a sufficient number of sample pairs were used to estimate the semivariance (Russo and Jury, 1987). Separation distances less than or equal to 16.2 m met or exceeded their 100-sample pair standard. When unidirectional semivariograms were constructed from the entire 490 sample data set, estimates of the range and integral scale were more than double those calculated from individual transects (Table 3). The Efaw forage N semivariogram had the longest range, 11.4 m, found in this analysis. The number of sample pairs used to identify this correlation distance was 192. All other semivariograms had more sample pairs. This analysis implies that transect sampling is an acceptable procedure for acquiring data for semivariance analysis, provided that the number of sample pairs used to construct the semivariogram exceeds 100 to 144 [on the basis of the Russo and Jury (1987) analysis].

Phosphorus at Efaw exhibited clearly defined multiple sills and ranges (Fig. 2d). There was a well-defined period of 3.5 to 4 m for unidirectional semivariance. This periodicity may be associated with agricultural machinery used on this farm. The data set had clusters of supports with large values of P. The major axes of these clusters were oriented parallel to a 2.13-m side of the area sampled. These axes paralleled the probable direction of tillage and fertilizer application. Although not as well defined, other Efaw semivariograms appeared to have similar length periods.

Ranges and integral scales corresponded closely for each soil and plant variable at both locations (Table 3, Columns 8 and 9). The two P integral scales were nearly

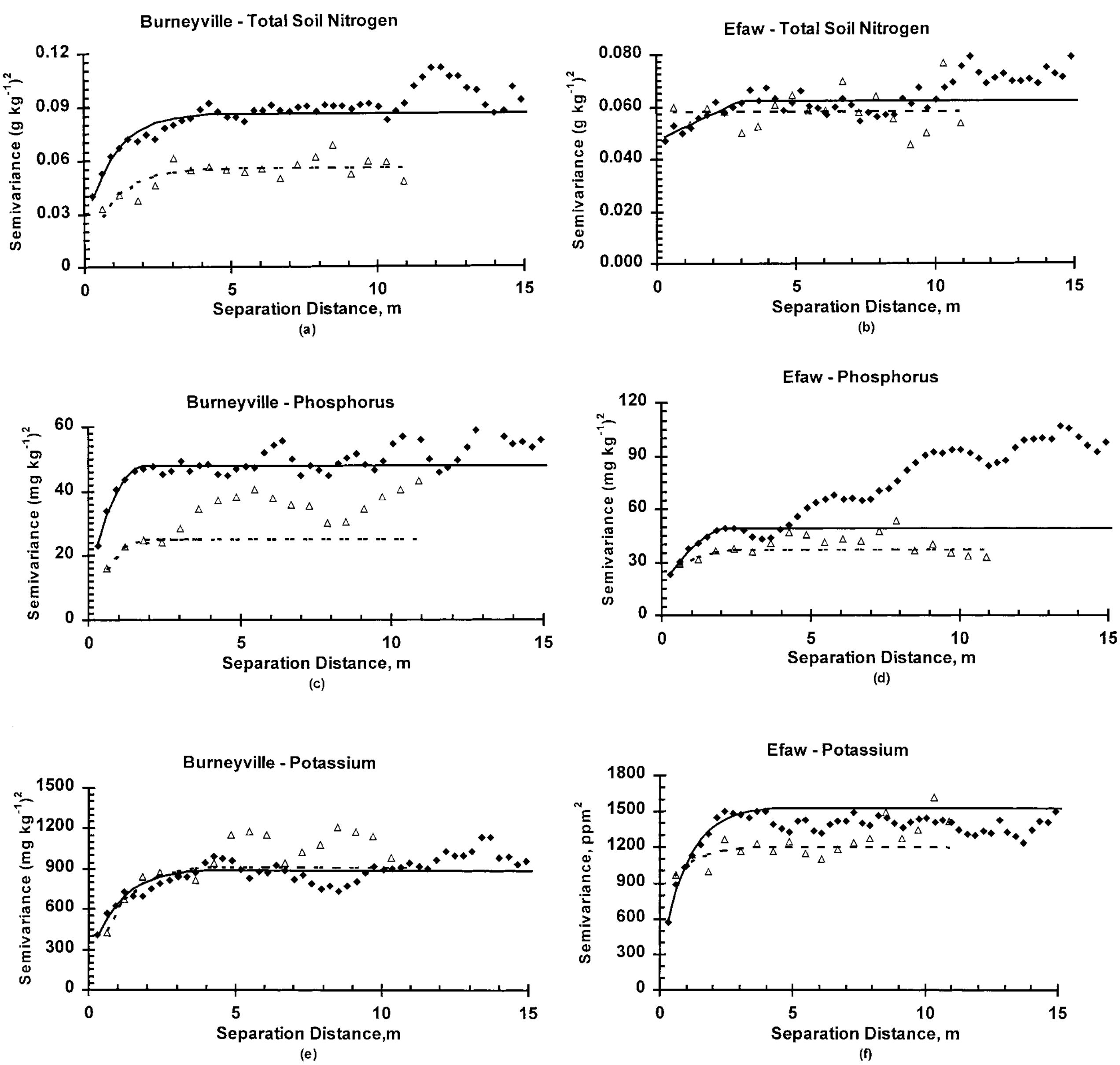

Fig. 2. Unidirectional semivariograms for 0.30 by 0.30-m supports (◆) and unidirectional semivariograms for 0.61 by 0.61-m supports (Δ) for five soil variables at two locations.

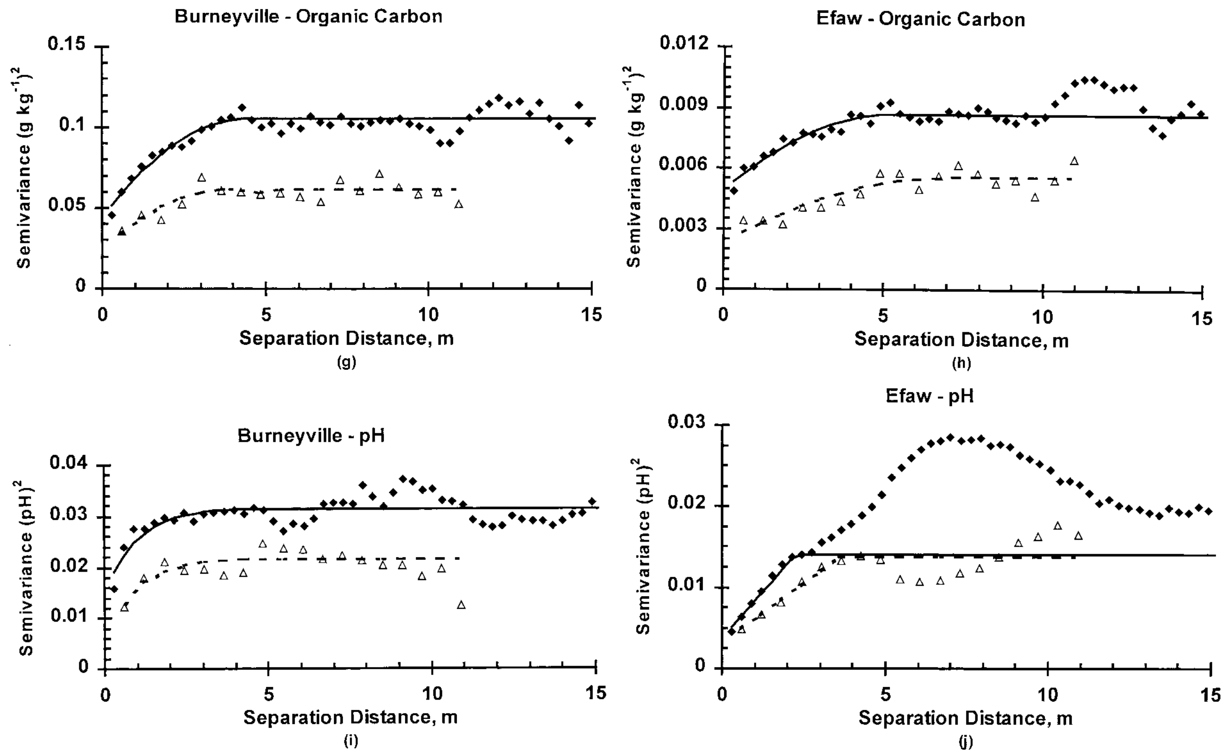

Fig. 2. Continued.

identical even though they were located on different soils, with one site grazed and the other site not grazed. Five 0.30- by 0.30-m sample supports (for all measured factors) had been deleted from the grazed Burneyville site because of unusually high P values which distorted the semivariogram. These values were likely caused by

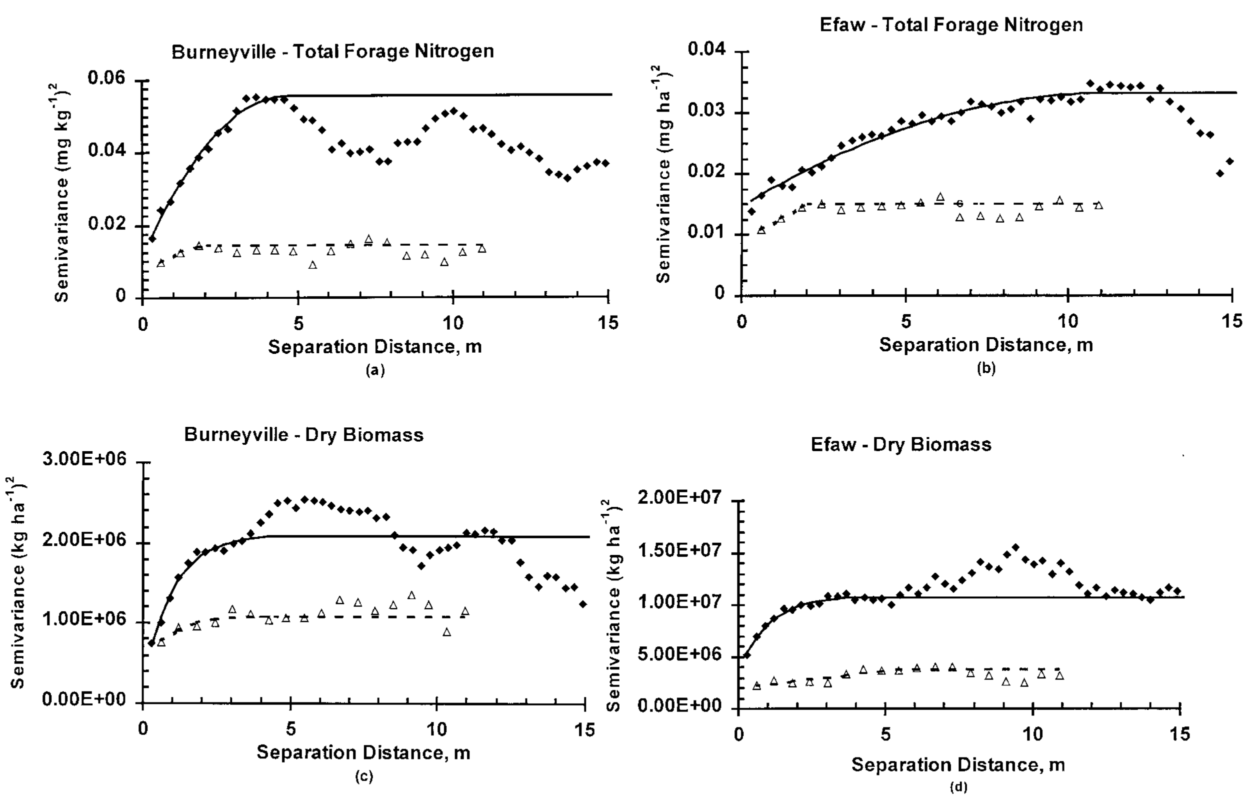

Fig. 3. Unidirectional semivariograms for 0.30- by 0.30-m supports (◆) and unidirectional semivariograms for 0.61- by 0.61-m supports (Δ) for two plant variables at two locations.

Table 3. Selected semivariance statistics for five soil and two plant variables at two locations calculated by transect and unidirectionally along the major axis of the experimental area with the entire data set. Support size is 0.30- by 0.30 m.

		21.34–m transects		Unidirectional				
Variable	Location	Median range	Median integral scale	Model	Nugget	Sill	Range	Integral scale
		m					m	
Soil N (g kg^{-1})	Burneyville	1.6	0.4	Exponential	0.0208	0.0860	4.3	0.7
	Efaw	<0.3	<0.3	Spherical	0.00449	0.00635	4.3	0.4
P (mg kg^{-1})	Burneyville	1.3	0.4	Spherical	14.2	48.0	1.9	0.5
	Efaw	1.5	0.4	Spherical	15.2	49.0	2.3	0.5
K (mg kg^{-1})	Burneyville	1.3	0.4	Linear-Sill	495	958	4.1	0.7
	Efaw	1.4	0.6	Exponential	251	1 529	4.4	0.8
Organic C (g kg^{-1})	Burneyville	2.6	0.7	Spherical	0.0429	0.1053	4.4	0.9
	Efaw	0.8	0.2	Spherical	0.0049	0.0087	5.3	0.8
pH	Burneyville	0.9	0.3	Exponential	0.0146	0.0313	4.0	0.5
	Efaw	2.3	1.0	Linear-Sill	0.0036	0.0141	2.2	0.8
Forage N (mg kg^{-1})	Burneyville	2.1	0.4	Spherical	0.0115	0.0557	4.6	1.4
	Efaw	5.1	1.4	Spherical	0.0147	0.0333	11.4	2.1
Biomass (kg ha^{-1})	Burneyville	1.7	0.4	Exponential	211 433	2 085 025	4.5	0.9
	Efaw	2.2	0.6	Exponential	3 463 809	10 729 394	4.2	0.6

cattle dung (Weeda, 1967; Castle and MacDaid, 1972). These results implied that a fundamental field element could be defined that applied to soils with different histories.

Regularized semivariograms calculated from the unit spherical transition model increased the semivariogram range, increased integral scale, and decreased sill magnitude when the support size was increased (Fig. 4 and Table 4). Projecting finite support semivariograms to zero separation distance is the recommended procedure for determining the nugget value of an experimental semivariogram. This procedure underestimated the value of the nugget, as was predicted by Journel and Huijbregts (1978). It must be noted that the nugget values were adjusted to the nugget value in Table 4 and Fig. 4. Equation [5] adjusts all semivariograms to zero nugget value. Since all soil samples are collected over a finite area, the authors' model assumed a nugget value of 0.302 for the *point* semivariogram. In this example, the magnitude of the range of the 0.30- by 0.30-m support was 7% greater than the point support. The magnitude of the integral scale of the 0.30- by 0.30-m support was 16% greater than point support. Results will be similar for other transition models.

Comparison of the sills and ranges of the 0.61- by 0.61-m support semivariograms with those of the 0.30- by 0.30-m semivariograms provided a second mechanism to investigate the effect of regularization on the semivariogram. Sill variances of all variables with 0.61- by 0.61-m supports were less than those of the 0.30- by 0.30-m support (Table 5), as was predicted by Eq. [6]. Journel and Huijbregts (1978) stated that deregularized ranges could be checked by regularizing their values over a larger support. All ranges for the 0.61- by 0.61-m semivariograms fell within the calculated upper and lower bounds of the regularized semivariograms with six exceptions. Two of these could not be compared: Efaw soil N, whose 0.61- by 0.61-m support semivariogram had a pure nugget effect and Efaw forage N 0.61- by 0.61-m semivariogram, whose longest separation distance was considerably less than the range of the 0.31- by 0.31-m semivariogram. Reregularization underestimated the range of Efaw organic C by 0.9 m, underestimated the range of Burneyville pH by 0.4 m, and underestimated Efaw biomass by 0.2 m. Only the reregularized range of Burneyville forage N was overestimated at 0.8 m. With the exception of Efaw organic C and Burneyville forage N, which had relatively large errors, estimating of the maximum and minimum possible values of the point support range by support geometry appears to be a reasonable procedure. In the case of the 0.30- by 0.30-m support geometry used in these experiments, the likely error in estimating the point support range will be ±0.3 m.

Field-Element Size Effect on Measurement and Application Error

One way of examining the effect of field-element size on error of the sensed field-element measurement from the true value of the measurement of the underlying fundamental field-element is to assume a sensor exists that can accurately and precisely measure the value of the sensed variable averaged over a small (<5 by 5 m) field-element. Research is being conducted to develop these sensors. Assuming that a sensor exists, the following question can be posed: What is the error from the measured value of an individual support of the sensor measured value for the field-element consisting of more than one support? As the field-element size decreases,

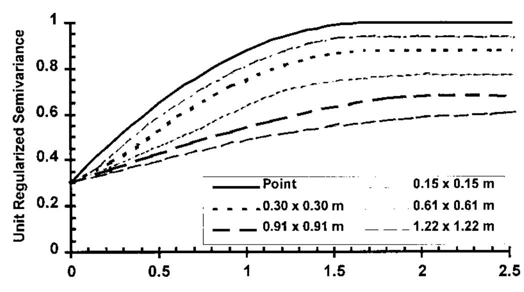

Fig. 4. Point and regularized semivariograms of a unit spherical transition model calculated with the equations presented in Journel and Huijbregts (1978).

Table 4. Effect of deregularizing from a 0.30- by 0.30-m support to a point and from a point to 0.61- by 0.61-m support on the upper and lower bounds of the semivariogram range, and 0.61- by 0.61-m support semivariogram statistics.

		0.61 × 0.61 m support			Regularized and deregularized range			
					Dereg. 0.30 × 0.31 m		Reg. 0.61 × 0.61 m	
Variable	Location	Nugget	Sill	Range	Lower limit	Upper limit	Lower limit	Upper limit
				m				
Soil N (g kg^{-1})	Burneyville	0.00662	0.00554	4.5	4.0	4.6	3.4	5.3
	Efaw	0.00562	–†	–	4.0	4.6	3.4	5.3
P (mg kg^{-1})	Burneyville	5.7	25.21	1.9	1.6	2.2	1.0	2.9
	Efaw	22.5	36.9	2.6	2.0	2.6	1.4	3.3
K (mg kg^{-1})	Burneyville	44.90	918	4.6	3.8	4.4	3.2	5.1
	Efaw	757	1 204	3.6	4.1	4.7	3.5	5.4
Organic C (g kg^{-1})	Burneyville	0.0612	0.0234	3.9	4.1	4.7	3.5	5.4
	Efaw	0.0023	0.0055	7.2	5.0	5.6	4.4	6.3
pH	Burneyville	0.0054	0.0216	4.3	3.7	4.3	3.1	5.0
	Efaw	0.0031	0.0137	3.6	1.9	2.5	1.3	3.2
Forage N (mg kg^{-1})	Burneyville	0.0049	0.0144	2.9	4.3	4.9	3.7	5.6
	Efaw	0.0091	0.0149	2.0	11.1	11.7	10.5	12.3
Biomass (kg ha^{-1})	Burneyville	502 807	1 086 198	4.0	4.2	4.8	3.6	5.5
	Efaw	1 975 923	3 733 115	5.4	3.9	4.5	3.3	5.2

† This semivariogram exhibited only a nugget effect.

Table 5. Effect of support dimensions on semivariogram statistics calculated for a unit spherical transition model.

Support	Nugget	Projected nugget	Sill	Range	Integral scale
				m	
Point	0.302	0.302	1.000	1.72	0.58
0.15 by 0.15 m	0.302	0.251	0.939	1.74	0.62
0.30 by 0.30 m	0.302	0.220	0.879	1.84	0.67
0.61 by 61 m	0.302	0.203	0.772	2.05	0.76
0.91 by 0.91 m	0.302	–†	0.680	2.27	0.85
1.22 by 1.22 m	0.302	–	0.602	2.51	0.95

† Not enough points were available to project a separate nugget.

the error between the values should approach zero. Support size in these experiments was 0.30 by 0.30 m. Calculations were performed to determine the mean errors between the measured values of each support within a field-element and the average value of that field-element (Table 6). Mean error decreased as much 50% as the field-element area measured decreased from the 6- by 60-support array to 2- by 2-support arrays. The greatest decrease occurred for variables exhibiting high CVs, P at Burneyville, and biomass at Burneyville and Efaw (Raun et al., 1998). The smallest decrease in error occurred with variables exhibiting low Cvs and pH at Burneyville and Efaw. For soil N at Efaw, the mean value of the error decreased from 10.3% for field-elements equal to the experimental area to 6.3% for field-element size of supports. Mean error could be very large, as for P at 46.2% when sensed over a 6- by 66-support field-element, or negligible, as for pH, 1.3% when sensed over the 2- by 2- or 4-support field-element. The greatest benefit occurred when reducing the field-element size for P, K and biomass. Reducing the field-element size for total soil N did not produce equivalent benefits, because the previous crop history suggested that total soil N would be uniformly distributed at comparatively low levels. Similar results occurred with dispersion variance, but the changes in magnitude were greater for certain variables such as P at Efaw.

An underlying assumption of any semivariance analysis is the existence of a fundamental field-element. Both the range and the integral scale have been used to define that area. Solie et al. (1996) concluded from semivario-

Table 6. Mean error from the true value of a 0.3 × 0.3 m element when the estimate of that value is the average of a 6 × 66 (1.8 × 19.8 m), 6 × 6 (1.8 × 1.8) m, 3 × 3 (0.9 × 0.9 m), and 2 × 2 (0.6 × 0.6 m) element array which includes that element and the dispersion variances of those arrays.

		Mean error				Dispersion variance			
Variable	Location	6 × 66	6 × 6	3 × 3	2 × 2	6 × 66	6 × 6	3 × 3	2 × 2
		%							
Soil N (g kg^{-1})	Burneyville	26.1	23.3	22.0	15.9	0.001125	0.001023	0.000925	0.000686
	Efaw	10.3	8.2	8.0	6.3	0.000136	9.08E-05	9.03E-05	7.38E-05
P (mg kg^{-1})	Burneyville	46.2	44.3	38.1	28.7	98.5	82.4	69.0	60.2
	Efaw	16.7	12.1	10.3	8.2	67.0	33.7	30.3	24.6
K (mg kg^{-1})	Burneyville	30.8	29.5	26.1	22.3	1 536	1 385	1 158	1 067
	Efaw	22.9	20.5	18.8	16.3	1 283	1 069	900	895
Organic C (g kg^{-1})	Burneyville	26.1	23.5	20.8	15.5	0.1297	0.1168	0.0967	0.0790
	Efaw	10.3	7.9	7.7	6.8	0.0218	0.0139	0.0134	0.0125
pH	Burneyville	3.1	2.7	2.8	2.2	0.0496	0.0386	0.0353	0.0331
	Efaw	2.3	2.0	1.5	1.3	0.0282	0.0156	0.0130	0.0116
Forage N (mg kg^{-1})	Burneyville	12.6	9.6	8.6	6.5	0.0511	0.0299	0.0246	0.0223
	Efaw	13.5	11.7	10.3	8.4	0.0318	0.0206	0.0183	0.0154
Biomass (kg ha^{-1})	Burneyville	44.5	44.8	38.1	29.8	2 633 327	2 372 474	2 026 083	1 675 790
	Efaw	49.2	43.1	34.5	30.0	527 448	411 333	354 518	338 837

gram analysis of optical sensor data that the fundamental field-element should be about 1.5 by 1.5 m. Another possible definition is the rooting area of the individual plants. The rooting distance of bermudagrass has not been reported, but the authors have observed pronounced striping when solution N fertilizer was applied with streaming nozzles spaced 0.25 m apart. For wheat, the rooting radius is approximately 0.3 m in the absence of competition (Lersten, 1987). If the definition of rooting radius is used to define field-element size, sensing 0.61- by 0.61-m area (2- by 2-support array) would produce no error in the value of the variable. Assuming competition among wheat plants reduces the rooting radius to 0.15 m, then sensing errors would fall between 1.3 and 30%. If the integral scale is accepted as the definition, the fundamental field-element size will be close to the unrestricted rooting diameter of wheat plant. A fundamental field-element size of 0.75 by 0.75 m appears to be acceptable, on the basis of the semivariance and field-element size error analyses.

Semivariance and error analyses of soil and plant data collected from 0.30- by 0.30-m supports at two locations clearly demonstrated the existence of integral scales whose magnitudes were less than 1 m. The regularized data collected in these experiments tended to overestimate the range and integral scale. The optimum field-element size for sensing purposes should be the integral scale unless the plant's root radius is larger. The experiments conducted here were not designed to determine the range and integral scale for large area variability. However, the existence of high levels of variability and relatedness at short distances raises the question at which level should variation of the soil or plant variable be treated. The degree of the meter-level variability in agricultural fields relative to the large area variability must be defined if the benefits of precision application of plant nutrients are to be optimized. In any case, to sense precisely the soil and plant variables measured in these experiments, measurements should be made at the meter or submeter level.

ACKNOWLEDGMENTS

The authors thank the Oklahoma Agricultural Experiment Station, the Soil Fertility Research and Education Advisory Board, The Potash and Phosphate Institute, and The Samuel Roberts Noble Foundation, Inc., Ardmore, OK, for supporting this work.

REFERENCES

Boyer, D.G., R.J. Wright, C.M. Feldhake, and D.P. Bligh. 1991. Soil spatial variability in steeply sloping acid soil environment. Soil Sci. 161:278–287.

Cahn, M.D., J.W. Hummel, and B.H. Brouer. 1994. Spatial analysis of soil fertility for site-specific crop management. Soil Sci. Soc. Am. J. 58:1240–1248.

Castle, M.D., and D. MacDaid. 1972. The decomposition of cattle dung and its effect on pasture. J. Br. Grassl. Soc. 27:133–137.

Chancellor, W.J., and M.A. Goronea. 1994. Effects of spatial variability of nitrogen, moisture, and weeds on the advantages of site-specific applications for wheat. Trans. ASAE. 37:717–724.

Follett, R.F., and G.A. Peterson. 1988. Surface soil nutrient distribution as affected by wheat-fallow tillage system. Soil Sci. Soc. Am. J. 52:141–147.

Gupta, R.K., S. Motaghimi, P.W. McClellan, M.M. Alley, and D.E. Brann. 1997. Spatial variability and sampling strategies for NO_3-N, P, and K determinations for site-specific farming. Trans. ASAE. 40:337–343.

Han, S., J.W. Hummel, C.E. Goering, and M.D. Cahn. 1994. Cell size selection for site-specific crop management. Trans. ASAE 37:19–26.

Hohn, M.E. 1988. Geostatistics and Petroleum Geology. Van Nostrand Reinhold, New York.

Isaaks, E.H., and R.M. Srivastava. 1989. An introduction to applied geostatistics. Oxford University Press, New York.

Journel, A.G., and C.H. Huijbregts. 1978. Mining geostatistics. Academic Press, New York.

Lersten, N. R. 1987. Morphology and anatomy of the wheat plant. p. 33–75. *In* E.G. Heyne (ed.) Wheat and wheat improvement. 2nd ed. Agon. Monogr. 13. ASA and SSSA, Madison, WI.

Matheron, G. 1989. Estimating and choosing: an essay on probability in practice. Springer-Verlag, New York.

Miller, M.P., M.J. Singer, and D.R. Nielson. 1988. Spatial variability of wheat yield and soil properties on complex hills. Soil Sci. Soc. Am. J. 52:1133–1141.

Nolin, M.C., S.P. Guertin, and C. Wang. 1996. Within-field spatial variability of soil nutrients and corn yield in a Montreal lowlands clay soil. p. 257–270. *In* P.C. Robert (ed.) Proc. of the third international conference on precision agriculture. Minneapolis, MN. 23–26 June 1996. ASA, CSSA, and SSSA, Madison, WI.

Raun, W.R., J.B. Solie, G.V. Johnson, M.L. Stone, R.W. Whitney, H.L. Lees, H. Sembiring, and S.B. Phillips. 1998. Microvariability in soil test, plant nutrient, and yield parameters in bermudagrass. Soil Sci. Soc. Am. J. 62:683–690.

Royle, A., I. Clark, P.I. Brooker, H. Parker, A. Journel, J.M. Rendu, R.I. Sandefur, and P. Mousset-Jones. 1980. Geostatistics. McGraw-Hill, Inc., New York.

Russo, D., and W.A. Jury. 1987. A theoretical study of the estimation of the correlation scale in spatially variable fields 1. Stationary fields. Water Resour. Res. 23:1257–1268.

Solie, J.B., W.R. Raun, R.W. Whitney, M.L. Stone, and J.D. Ringer. 1996. Optical sensor based field element size and sensing strategy for nitrogen application. Trans. ASAE 39:1983–1992.

Solie, J.B., K.P. Self, and J.D. Summers. 1987. Factors affecting low volume spray patterns of a floatation sprayer. Trans. ASAE 30:1631–1636.

Solie, J.B., R.W. Whitney, and M.F. Broder. 1994. Dynamic pattern analysis of two pneumatic granular fertilizer applicators. Appl. Eng. Agric. 10:335–340.

SPSS. 1997. TableCurve 2D. V.4. SPSS Inc., Chicago.

Stone, M.L., J.B. Solie, W.R. Raun, R.W. Whitney, S.L. Taylor, and J.D. Ringer. 1996. Use of spectral radiance for correcting in-season fertilizer nitrogen deficiencies in winter wheat. Trans. ASAE 39:1623–1631.

Weeda, W.C. 1967. The effect of cattle dung patches on pasture growth, botanical composition, and pasture utilization. N.Z. J. Agric. Res. 10:150–159.