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The economic potential of precision nitrogen application with wheat based on plant sensing

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Abstract

Plant-based precision nitrogen fertilizer application technologies have been developed as a way to predict and precisely meet nitrogen needs. Equipment necessary for precision application of nitrogen, based on sensing of growing wheat plants in late winter, is available commercially, but adoption has been slow. This article determines the expected profit from using a plant-sensing system to determine winter wheat nitrogen requirements. We find that plant-sensing systems have the potential to be more profitable than traditional nonprecise systems, but the existing system simulated was roughly breakeven with a traditional system.

JEL classification: Q12

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1. Introduction

Past research suggests that most agricultural producers apply more nitrogen than is needed in most years. Producers make nitrogen fertilization decisions in the face of both spatial and temporal uncertainty; that is, they must decide how much nitrogen fertilizer to apply in a particular field and year knowing that plant nitrogen needs vary substantially within the same field, across fields, and across growing seasons. If crop producers had accurate information about how much nitrogen is needed on each portion of the field in a given year, they could reduce fertilizer costs. Applying only the amount of nitrogen necessary for plants to reach their yield potential could also reduce nitrate contamination in groundwater since nitrogen not used by plants may leach into groundwater.

Precision application of nitrogen based on soil sampling and yield monitors has been developed to help producers decide how much nitrogen to apply. However, costs and measurement errors have limited the usefulness of nitrogen recommendations based on yield monitors and soil sampling of small grids (Arslan and Colvin, 2002; Babcock et al., 1996; Bullock et al., 2009). Soil test measurements for phosphorus and potassium levels have been shown to be more reliable than soil tests for nitrogen levels; however, precision soil sampling with wheat has not even proven profitable for these nutrients (Kilian, 2001; Lowenberg-DeBoer and Aghib, 1999; Swinton and Lowenberg-DeBoer, 1998). The use of yield monitors to fine tune nitrogen application has also been limited because while yields vary substantially across the field, they do not vary in the same way every year (Asim, 2000). These limitations, associated with the use of soil sampling and yield monitors, might explain why few producers use these technologies to determine how much nitrogen to apply (Daberkow and McBride, 2000). Plant sensing, however, is in-season and can give much more accurate predictions of nitrogen needs.

Sensing can be done from satellites, airplanes, fertilizer applicators, or from hand-held devices. The focus here is on the latter two methods. The work here uses the normalized difference vegetative index (NDVI), which is based on a near-infrared sensor (Mullen et al., 2003; Phillips et al., 2004; Raun et al., 2002, 2005), but other indices and sensors have been used (Alchanatis et al., 2005; Begiebing et al., 2007; Ehlert et al., 2004; Havránková et al., 2007). Plant sensing is promising since it is a more direct measure of nitrogen needs than soil

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Data Appendix Available Online

A data appendix to replicate main results is available in the online version of this article. Please note: Wiley-Blackwell, Inc. is not responsible for the content or functionality of any supporting information supplied by the authors. Any queries (other than missing material) should be directed to the corresponding author for the article.

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tests and yield monitors. However, adoption of such technologies has been slow (Lowenberg-DeBoer, 2006). Plant sensing is clearly an outstanding technical achievement, but it apparently faces some economic hurdles. One economic challenge to plant sensing is that it requires the use of higher priced sources of nitrogen fertilizer (e.g., liquid urea ammonium nitrate (UAN) solutions) relative to the conventional pre-plant source of nitrogen commonly used in the region (i.e., anhydrous ammonia (NH_3)). As a result, the gains in nitrogen use efficiency (NUE) of the sensor-based system must be large enough to offset the substantial cost savings associated with using the lower priced anhydrous ammonia.

Plant-sensing technology requires investment in specialized plant-sensing equipment that once purchased, farmers and fertilizer firms cannot easily resell to recoup the investment. When investment is irreversible, there is an option value in postponing the decision to invest and search for new information. However, accurate information about expected producer benefits from using plant sensing is lacking. This lack of information may explain why adoption has been slow (Isik et al., 2005). Information about economic performance of plant-sensing technologies could also be valuable to agricultural machinery manufacturers and fertilizer companies since it could provide them with a target cost needed to encourage producers to adopt the technologies.

The objective of this research is to determine the potential profitability of nitrogen recommendations based on whole-field and variable-rate winter wheat plant sensing relative to conventional practices. Data obtained from field trials are used to estimate expected returns from five nitrogen fertilization alternatives for winter wheat that is planted in the late summer or early fall and harvested the following June. The five systems include (1) a check system with no nitrogen added either pre-plant or late winter; (2) a pre-plant NH_3 uniform rate of 90 kg per ha of nitrogen, a proxy for the state extension service recommendation and current farmer practice; (3) a late-winter topdress UAN application with a uniform rate based on plant sensing; (4) a late-winter topdress UAN precise-rate system based on real-time sensing (perfect knowledge); and (5) a late-winter topdress UAN precise-rate system based on real-time sensing and a nitrogen fertilizer optimization algorithm (NFOA) developed by Raun et al. (2002). There are several commercially available real-time sensors for nitrogen fertilization (Ehlert and Dammer, 2006). The focus here is on the Raun et al. (2002) system that is marketed under the name Greenseeker since it is specifically calibrated for winter wheat production in the Southern Plains of the United States. We develop a yield response to nitrogen function that is conditional on plant sensing. A difficulty that had to be overcome is that for the available experimental data, all nitrogen was applied pre-plant.

2. Theory

Farmers must decide prior to fall planting whether or not they want to use the plant-sensing technologies considered here. While the decision is made under uncertainty, the choice of fertilization strategy is likely to have a small effect on the profit variability of the entire farm; therefore an expected profit-maximizing framework is appropriate. The decision rule for the risk-neutral producer can be written as

$$\text{Producer decision} = \begin{cases} \text{adopt}, & \text{if } E(\max E(\pi^{\text{new}})) - E(\max E(\pi^{\text{old}})) > \lambda, \\ \text{not adopt}, & \text{otherwise}, \end{cases} \tag{1}$$

where $\lambda \geq 0$ represents the cost of change and $E(\pi^k)$ is the expected profit from using the kth technology ($k = \{\text{new, old}\}$). As Eq. (1) shows, if the expected profit from the proposed technology is greater than the expected profit with the traditional technology plus the cost of change, then the new technology will be adopted. The plant-sensing technology considered here may provide economic value from increasing grain, reducing costs, or some combination of both.

The plant-sensing system of Raun et al. (2002) essentially requires that a producer conduct a nitrogen response experiment in each field. The experiment can consist of a single nitrogen-rich strip (NRS) where a nonlimiting level of nitrogen is applied pre-plant so that nitrogen will not be the constraining input, allowing plants to reach their yield plateau. But, it can also consist of a set of ramped nitrogen strips where increasingly higher amounts of nitrogen are applied. With an NRS experiment, sensors are used to compare the relative growth between the plants growing in the NRS and the plants growing in the unfertilized areas of the field. A formula is then used to determine nitrogen needs in the unfertilized areas of the field. With the ramped strips technique, a linear plateau model can be estimated and the nitrogen level to reach the plateau is the recommended level so long as the marginal value product of the yield at the plateau is greater than the marginal input cost. Nitrogen needs can vary across the field and real-time systems have been developed to sense and fertilize grids smaller than a square meter in an attempt to apply just the right amount of nitrogen to each grid.

We assume here that the nitrogen application system chosen and the timing of N application do not affect the optimal quantity of other inputs. Nitrogen can either be applied pre-plant, in which case NH_3 can be knifed into the soil, or nitrogen can be applied as a topdress application (here assumed to be liquid 28% UAN) in late winter to growing plants. Assuming that price and yield are uncorrelated, the producer's optimization problem can be represented as

$$\begin{aligned} &\max_{N^P, N^T, \lambda} \mathrm{E}(R) = p\mathrm{E}(y) - r^P N^P - r^T N^T - \lambda_1 C^P \\ &\quad -\lambda_2 C^{\text{NRS}} - \lambda_3\left[(1-\lambda_2)C^T + \lambda_2 C^{N(ORI)}\right], \\ &\text{s.t.} \\ &y = y(N), \\ &N = N^P + \psi N^T, \\ &\text{If } N^P > 0 \text{ then } \lambda_1 > 0 \\ &\text{If } N^T > 0 \text{ then } \lambda_3 > 0 \\ &\text{If } \lambda_2 > 0 \text{ then } N^T = N(ORI) \\ &\lambda_i \in \{0,\ 1\} \quad \forall i, \quad \text{and} \\ &N^{\mathrm{P}},\ N^T \geq 0, \end{aligned} \tag{2}$$

where R is the net return above nitrogen fertilizer application costs; y is the yield; N is the sum of pre-plant nitrogen (N^P) and topdress nitrogen (N^T); $\psi \geq 1$ is the relative efficiency of topdress nitrogen (UAN) relative to pre-plant (NH_3); p represents the expected price of wheat; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3)$ is a vector of binary choice variables; r^P and r^T represent the prices of pre-plant nitrogen and topdress nitrogen, respectively; C^P represents pre-plant nitrogen application costs; C^{NRS} represents the cost of the NRS; $C^{N(ORI)}$ represents the cost of a machine that senses the plants with optical sensing where the optical reflectance index (ORI), which is based on NDVI, measures the amount of nitrogen available to the plants at the time of sensing and then applies a precise quantity of topdress nitrogen to each grid; C^T represents the conventional topdress nitrogen application costs; and the function $N(ORI)$ is the application rate algorithm based on precision-sensing information (NRS). Note that λ_3 is selected conditional on NRS being known.

Increased yields with precision plant sensing could come about from conventional systems applying either too much or too little nitrogen. The evidence regarding whether excess nitrogen causes yields to decline is mixed (Biermacher et al., 2006) but tends to suggest a little or no yield decrease from applying excess nitrogen. A conventional system that applied too little nitrogen would clearly lead to lower yields than a precision-sensing system that applied exactly the amount of nitrogen needed. In practice however, wheat producers in the region apply more nitrogen than is needed in most years (Hossain et al., 2004). As a result, most of the advantage of precision sensing for winter wheat producers is expected to be due to the reduced cost of nitrogen fertilizer rather than the increased yield. Although agronomic results (Raun et al., 2002 and Mullen et al., 2003) clearly show that the optical-sensing system uses substantially less total nitrogen, it faces a major economic challenge because plant sensing uses nitrogen in liquid form (UAN), which is more expensive than anhydrous ammonia (NH_3) used with conventional technologies (i.e., $r^P < r^T$).

3. Procedures

The stochastic plateau function of Tembo et al. (2008) was developed specifically to match the production function assumed by Raun et al. (2002) and has been used successfully by Kaitibie et al. (2003, 2007) Therefore, we use Tembo et al.'s linear response stochastic plateau function to represent wheat response to nitrogen:

$$y_{it} = \min\left[\beta_0 + \beta_1 ORI^S_{it}\left(N^P_{it}\right) + \beta_2 N^T_{it}, \mu_m + v_t\right] + u_t + \varepsilon_{it}, \tag{3}$$

where y_{it} is the wheat yield in kg per ha on grid i in year t; N^P_{it} is the level of pre-plant nitrogen; N^T_{it} is the level of topdress nitrogen; ORI^S_{it} (N^P_{it}) represents an optical reflectance index taken in the late winter on grid i in year t; μ_m is the average plateau yield, β_0, β_1, and β_2 are the parameters to be estimated; v_t represents the plateau year random effect; u_t is a year random effect that shifts the intercept, and ε_{it} is an i.i.d. normal error term.

Our data include pre-plant nitrogen and ORI readings for pre-plant nitrogen, but no topdress nitrogen was applied. The available data were collected for another purpose. Therefore, Eq. (3) cannot be estimated directly. To circumvent this limitation, we assume that the marginal productivity of topdress nitrogen is the same as (or at least proportional to) the marginal productivity of pre-plant nitrogen. Next, we estimate two separate regressions: wheat yield is regressed on the ORI, and the ORI is regressed on pre-plant nitrogen. Estimates from these regressions are then used to construct Eq. (3).

Let the relationship between wheat yield and the ORI be written as

$$y_{it} = a + bORI^S_{it}\left(N^P_{it}\right) + \theta_{it}, \tag{4}$$

where y_{it} is the wheat yield in kg per ha on grid i in year t, a and b are the intercept and slope coefficients to be estimated respectively, and the error term θ_{it} is partitioned into an independently and identically distributed random error term η_{it} that has mean zero and variance σ^2_η, and the year random effect ω_t that has mean zero and variance σ^2_ω. We use a linear function rather than an exponential function like Raun et al. (2002) because it allows deriving an analytical solution. The estimated exponential model is very close to linear so this difference is relatively unimportant.

Independence is assumed between the two variance components, and therefore the variance of the overall error term is $\sigma^2_\theta = \sigma^2_\omega + \sigma^2_\eta$. The symbol ORI^S_{it} (N^P_{it}), previously defined as the ORI taken in the late winter on grid i in year t, is the normalized difference vegetation index sensor reading divided by the number of growing degree days. The ORI measures the amount of nitrogen available to the plants at the time of sensing, which in turn helps in quantifying the amount of additional nitrogen needed to reach plateau yields.

The second regression used to construct Eq. (3) is the regression of ORI on pre-plant nitrogen. This relationship is defined as

$$ORI^S_{it}\left(N^P_{it}\right) = \min\left(\alpha + \beta N^P_{it}, ORI^M + v_t\right) + u_t + \varepsilon_{it}, \tag{5}$$

where ORI^S_{it} (N^P_{it}) is as defined previously; α and β are the intercept and slope parameters to be estimated respectively; N^P_{it} is nitrogen applied on grid i at the time of planting in year t; ORI^M is the expected plateau level of ORI; $v_t \sim N(0, \sigma^2_v)$ represents year random effects on the plateau; $u_t \sim N(0, \sigma^2_u)$ represents year random effects; $\varepsilon_{it} \sim N(0, \sigma^2_\varepsilon)$ is the traditional random error term. The null hypothesis that the response part of Eq. (5) is linear was tested versus the alternative of a quadratic response. The null hypothesis of a linear model could not be rejected using a likelihood ratio test (the actual chi-squared value was 3.3 and the 0.05 critical value is 3.8).

The estimates from Eqs. (4) and (5) are used to construct Eq. (3). Again, the key assumption is that the marginal productivity of topdress nitrogen is the same as (or at least proportional to) the marginal productivity of pre-plant nitrogen; that is, $\beta_1 = b$, and $\beta_2 = b\beta$. When the plant-sensing technology is used to obtain a single estimate of the uniform whole-field nitrogen requirement, only an average measure of ORI is used, which implies that the spatial variation on each grid remains. However, since sensor measurements are taken from the NRS, no error in measuring the plateau is assumed. With this assumption we set $\beta_0 = a$ from Eq. (4), and Eq. (3) can be re-written as

$$y_{it} = \min\left[a + b\left(\alpha + \beta N_{it}^{P}\right) + b\beta N_{it}^{T}, a + bORI^{M} + bv_t\right] + bu_t + b\varepsilon_{it} + \theta_{it}, \tag{6}$$

which imposes $\partial y_{it}/\partial N_{it}^{T} = \partial y_{it}/\partial N_{it}^{P} = b\beta$.

Determining the optimum pre-plant level of nitrogen analytically using the stochastic plateau model in Eq. (6) is not straightforward because year and spatial random effects enter Eq. (6) nonlinearly. The optimal level of nitrogen to apply with this functional form has been developed by Tembo et al. (2008). The optimum input level ($N_{it}^{P^*}$) can be determined as

$$N_{it}^{P^*} = \min\left(0, \frac{1}{\beta}\left(ORI_t^{M} + Z_\delta \sigma_v - \alpha\right)\right), \tag{7}$$

where Z_δ is the critical Z-value where $\delta = 1 - \Phi = r/(pb\beta)$ is the observed probability in the right-hand tail of the $N(0, 1)$ distribution, r is the price of nitrogen, and p is the price of wheat.

If the variable rate plant-sensing technology is applied, and we assume information from the NRS and each grid is sensed perfectly, then we can re-write Eq. (3) as

$$y_{it} = \min\left[a + bORI_{it}^{S}\left(N_{it}^{P}\right) + b\beta N_{it}^{T}, a + bORI_t^{\mathrm{NRS}}\right] + \theta_{it}, \tag{8}$$

where ORI_t^{NRS} is the in-field experimental measure from an NRS. The model in Eq. (8) is a linear plateau model and the optimum is the level of nitrogen needed to reach the plateau on each grid, which is

$$N_{it}^{T^*} = \begin{cases} \dfrac{ORI_t^{\mathrm{NRS}} - ORI_{it}^{S}}{\beta}, & \text{if } pb\beta > r, \\ 0, & \text{otherwise.} \end{cases} \tag{9}$$

Note that we are implicitly assuming that none of the errors in Eq. (5) represents measurement error. If we were to add measurement error, we would end up with the model developed by Berck and Helfand (1990) and Paris (1992). Adding measurement error would further reduce the value of sensing.

When the sensor-based system is used to obtain a uniform, whole-field N application recommendation, the ORI on each individual grid in the field is not measured. Instead, only a sample of sensor measurements is collected, providing for an average measure of ORI. This implies that spatial variation on each grid is expected to be present. However, since sensor measurements are taken from the NRS, which covers a strip across the length of the field, no error in the plateau is assumed. This implicitly assumes that all variation across grids is due to differences in available nitrogen and therefore all grids have the same yield plateau (which is likely not true in practice and is yet another assumption that causes our result to favor sensing). The error remains in the response portion of the plateau and the production function becomes

$$y_{it} = \min\left[a + b\overline{ORI}_{it}^{S} + b\beta N_{it}^{T} + b\varepsilon_{it}, a + bORI_t^{\mathrm{NRS}}{}_t\right] + \theta_{it}, \tag{10}$$

where $\overline{ORI}_{it}^{S}$ is an average ORI reading across an unfertilized portion of the field near the NRS. The solution to the optimal level of nitrogen in Eq. (10) is analogous to Eq. (6) except that the upper rather than the lower tail of the distribution is needed. The optimal whole-field uniform rate ($N_{it}^{W^*}$) can be determined as

$$N_{it}^{W^*} = \min\left(0, \frac{1}{\beta}\left(ORI_t^{M} + Z_{\delta_W}\sigma_\varepsilon - \overline{ORI}_{it}^{S}\right)\right), \tag{11}$$

where Z_{δ_W} is the critical Z-value where $\delta_W = \Phi = r/(pb\beta)$ is the observed probability in the right-hand tail of the $N(0, 1)$ distribution, r is the price of nitrogen, and p is the price of wheat. Note that the current agricultural extension service recommendation for whole-field uniform rates in the region will underestimate the optimal level of nitrogen because it does not consider the remaining spatial variability. Also note that in an actual field, the plateau might vary across grids, which again is a simplification that could cause the value of sensing to be overstated, unless the sensing could also identify grids with less yield potential (research under way attempts to do so by not applying nitrogen where there are few or no plants). Other types of measurement error can be readily included in the same way as the spatial variability is included.

3.1. Optimizing nitrogen using the nitrogen fertilizer optimization algorithm (NFOA)

The NFOA developed by Raun et al. (2002) is used to determine how much topdress nitrogen is needed. Note that this system is disadvantaged here because it assumes a higher marginal product of nitrogen than the function simulated. Following Raun et al., the optimal level of nitrogen is defined as

$$N_{it}^{\mathrm{NFOA}} = \tau(YPN_{it} - YP0_{it})/\gamma, \tag{12}$$

where τ is 0.0239, since on average wheat contains 2.39% nitrogen, γ is a constant that represents the level of NUE that is the percentage of nitrogen that is used by the plant rather than lost (we use the midpoint of the range recommended by Raun et al. (2002) who recommend using an NUE between 0.50 and 0.70), $YP0_{it}$ is the yield response to ORI and gives an estimate at the time of sensing for wheat yield potential when no additional nitrogen is added. In the NFOA, $YP0_{it}$ is defined mathematically as

$$YP0_{it} = c_0 \exp(ORI_{it} c_1), \tag{13}$$

where c_0 and c_1 are the intercept and the slope parameters.[1] The symbol ORI_{it} denotes the ORI taken in the late winter on grid i in year t, and the symbol YPN_{it} in Eq. (12) is defined as the yield potential when additional nitrogen fertilizer is applied in the late winter at a level necessary to bring plant growth to the maximum potential, or mathematically:

$$YPN = \begin{cases} \max((RI \times YP0), YP0), \\ \qquad\qquad \text{if } \max((RI \times YP0, YP0) < y^{\max}), \\ y^{\max}, \quad \text{otherwise}, \end{cases} \tag{14}$$

where RI is a response index that is calculated as

$$RI = \frac{ORI \text{ from NRS}}{ORI \text{ from farmer practice}} = \frac{ORI_t^{\text{NRS}}}{E(ORI_{it}^S)} = \frac{ORI_t^{\text{NRS}}}{\alpha + u_t}. \tag{15}$$

According to Raun et al. (2002), $y^{\max}$ is the biological maximum yield for the specific crop grown within a specific region and under defined management practices. Raun et al. use 6,988 kg per ha for the maximum yield of dryland winter wheat produced in central Oklahoma. Substituting Eq. (14) into Eq. (15) gives

$$YPN_{it} = \begin{cases} \max\left(\left(\dfrac{ORI_t^{\text{NRS}}}{\alpha + u} \times 0.359\exp(ORI_{it} \times 0.3244)\right),\right. \\ \qquad\qquad \left. 0.359\exp(ORI_{it} \times 0.3244)\right), \text{ if} \\ \max((RI \times YP0, YP0) < y^{\max}), \\ y^{\max}, \text{ otherwise}. \end{cases} \tag{16}$$

Noting that the response function we have estimated in this paper is modeled after the NFOA is important. The main difference is that we estimate the marginal product from the data rather than based on agronomic relationships.

4. Data and empirical procedures

The parameters of Eqs. (4) and (5) are estimated using data from nine years of on-farm winter wheat experiments conducted either on or near agronomic research stations throughout Oklahoma from 1998 to 2006. The locations are near Perkins, Stillwater, Hennessey, Haskell, Tipton, and Lahoma. Most of the locations are distant from each other so site-years are used rather than years in the estimation of random effects. As Bullock et al. (2009) note, the use of several years of data and multiple locations is important in precision agriculture research, but most research including Bullock et al. (2009) have either had only one or two years of data or one or two locations. These data include observations for wheat yield, ORI, and pre-plant nitrogen applied. All nitrogen was applied pre-plant.

Varying levels of nitrogen were applied to each experimental plot prior to planting wheat in late September or early October. All ORI readings were taken during Feekes growth stages 4 (leaf sheaths beginning to lengthen) and 5 (pseudo-stem, formed by sheaths of leaves strongly erect) (Large, 1954). ORI readings were collected from a 4 square meter area between 10 a.m. and 4 p.m. under natural lighting between January and March. Grain yield was measured from the same 4 square meter area. The mean ORI reading on plots that received no nitrogen was 5.60, while the average ORI on the NRS was 7.10, which shows that the ORI was able to capture nitrogen stress. The cell means are presented in Table 1. Table 1 illustrates that nitrogen was usually beneficial, but not always. Additional information regarding the experiments can be found in Mullen et al. (2003).

Anselin et al. (2004) explain that spatial autocorrelation can take the form of a spatial lag or a spatial error model. With a spatial lag model, the level of nitrogen applied to a plot would affect the yield on an adjacent plot. The agronomists only harvest the center of the plots so spatial lag effects are unlikely although not impossible since nitrogen is mobile in the soil. Spatial error models are common in farmer's fields. Spatial error models are created when there are two or more distinctly different areas of a field such as areas with two different soil types. The data used here are collected from plots that were placed in areas that were selected to be uniform and so spatial error is likely small. If it had been possible to estimate a spatial error model in this research, we can expect that it would have had its usual result of slightly reducing t-values and having a little effect on coefficient estimates. Future research may want to consider estimating a stochastic plateau model in the presence of spatial lag or spatial error effects. A greater concern than spatial autocorrelation, however, is that the data used here may have less spatial variability than farmers' fields.

Our measures of the value of grid sensing apply to the size of plots that we have in our dataset. One-square-meter plots would have additional variability and thus the value of sensing smaller plots would be larger than that estimated here. All plots are near each other. In an on-farm situation, some plots would be farther from the NRS than with the data used here. If the field were not uniform, predictions would be less accurate in an actual on-farm situation than with the data used here.

Carryover of nitrogen is usually small, but there is a potential effect of carryover of excess nitrogen that is not considered. Raun et al. (1998) find some greater carryover of nitrogen on the plots with the largest nitrogen applied in long-term experiments. Some of the plots used here are from long-term experiments. If carryover is larger on plots with more applied N, ignoring the carryover will bias the estimates of our stochastic plateau model

[1] Note that parameter estimates have been shifted one standard deviation out to the left in an effort by Raun et al. (2005) to describe a yield frontier. Current estimates of c_0 equal to 0.359 and c_1 equal to 0.3244 describe the frontier.

Table 1
Wheat grain yield by N application level, location and year (kg/ha)

	Locations of agronomic experiments								
Year	L1	L2	L3	L4	L5	L6	L7	L8	L9
1998	na (0)	na (0)	na (0)	na (0)	na (0)	1,146 (0) [20.0, 9]	1,313 (0) [18.4, 12]	na (0)	3,028 (0) [11.5, 12]
	na (56)	na (112)	na (56)	na (22)	na (45)	1,601 (56) [22.1, 9]	1,390 (56) [25.6, 12]	na (45)	3,669 (56) [11.5, 12]
	na (90)	na (168)	na (90)	na (45)	na (90)	2,061 (112) [22.8, 9]	1,449 (112) [23.9, 12]	na (90)	4,009 (112) [11.7, 12]
	na (123)		na (123)	na (67)		2,194 (168) [24.2, 9]	1,486 (168) [34.6, 12]	na (123)	5,276 (168) [6.2, 12]
				na (90)					
				na (112)					
1999	2,167 [8.1, 3]	1,766 [23.9, 8]		1,563 [15.2, 8]		1,076 [33.7, 3]		855 [23.2, 8]	
	3,246 [12.1, 6]	2,311 [40.1, 15]		2,077 [17.7, 4]		1,788 [25.4, 3]		1,344 [12.2, 4]	
	3,510 [7.6, 6]	1,581 [32.3, 4]		2,599 [18.9, 4]		2,566 [12.9,3]		1,722 [11.81]	
	3,513 [13.6, 6]			3,196 [12.8, 4]		2,292 [11.1, 3]		1,695 [44.6, 3]	
				3,983 [16.9, 4]					
				4,439 [3.72, 4]					
2000	2,182 [5.8, 3]	2,490 [5.8, 3]	3,797 [2.9, 3]	1,897 [16.1, 8]		2,603 [na, 3]		1,176 [14.6, 6]	
	2,298 [16.5, 6]	2,936 [26.4, 14]	3,992 [9.0, 6]	2,682 [10.1, 4]		2,947 [27.2, 3]		1,850 [15.4, 4]	
	2,547 [38.8,6]	1,627 [35.5,4]	3,564 [26.1,6]	2,974 [16.1,4]		3,894 [5.7,3]		2,448 [12.3,4]	
	2,514 [39.2, 6]		3,900 [11.6, 6]	3,521 [9.2, 4]		2,981 [35.7, 3]		3,609 [3.8, 4]	
				3,540 [9.1, 4]					
				3,476 [30.4, 4]					
2001	2,691 [9.3, 3]	3,119 [27.2, 8]				2,748 [6.7, 3]		1,053 [31.1, 8]	
	3,184 [12.6,6]	3,034 [43.1,15]				2,372 [10.9,3]		1,622 [40.7,4]	
	3,335 [24.2, 6]	2,529 [19.4, 4]				2,287 [24.8, 3]		1,942 [14.3, 4]	
	3,263 [13.2, 6]					2,496 [25.5, 3]		1,328 [25.0, 3]	
2002	1,560 [36.7, 2]	3,110 [16.9, 2]	2,884 [22.9, 3]	2,709 [18.2, 8]		2,923 [34.3, 3]		1,060 [15.7, 6]	
	2,927 [9.9, 6]	1,932 [48.0, 16]	2,933 [19.1, 6]	3,035 [na, 1]		2,588 [6.6, 3]		1,630 [20.1, 4]	
	3,774 [15.1,6]	1,149 [46.3,4]	3,134 [26.3,6]	2,950 [24.1,3]		2,618 [14.4,3]		2,421 [13.9,4]	
	3,370 [21.4, 6]		2,784 [37.5, 5]	2,709 [23.7, 3]		3,249 15.7, 3]		2,985 4.3, 4]	
				2,658 [13.9, 2]					
				2,928 [5.7, 3]					
2003	2,945 [16.8, 3]			2,714 [14.9, 8]		2,794 [6.2, 3]		1,357 [11.8, 2]	
	3,719 [22.9, 6]			4,034 [16.4, 4]		3,512 [11.6, 3]		1,951 [19.2, 4]	
	4,739 [23.1, 6]			4,750 [13.0, 4]		3,814 [12.0, 3]		2,092 [37.8, 4]	
	4,946 [11.9, 6]			4,901 [8.1, 3]		3,738 [13.2, 3]		2,686 [23.7, 4]	
				6,107 [18.2, 3]					
				5,956 [2.2, 3]					
2004				1,476 [23.4, 8]	2,426 [17.6, 4]	1,935 [16.2, 3]		1,516 [25.7, 8]	
				1,765 [25.6, 4]	2,743 [17.2, 4]	3,042 [33.5, 3]		2,145 [25.9, 4]	
				1,936 [32.2,4]	4,271 [17.7,4]	3,775 [18.3,3]		2,910 [30.4,4]	
				3,216 [23.9, 4]		3,019 [28.4, 3]		1,295 [30.8, 4]	
				3,100 [25.5, 4]					
				3,412 [10.9, 4]					
2005				1,694 [20.4, 8]		3,354 [18.9, 3]		1,770 [31.6, 8]	
				2,113 [13.5,4]		4,057 [21.5,3]		1,933 [18.2,4]	
				2,607 [15.1, 4]		4,271 [10.7, 3]		2,452 [3.9, 4]	
				2,767 [10.7, 4]		3,735 [10.3, 3]		2,273 [13.9, 4]	
				2,476 [44.1,4]					
				2,806 [29.9, 4]					
2006				2,398 [22.5, 8]	1,276 [56.1, 4]	917 [22.4, 3]		891 [16.2, 8]	
				2,498 [24.6, 4]	3,175 [18.1, 4]	1,608 [15.9, 3]		764 [37.9, 4]	
				3,109 [4.7, 4]	4,373 [16.4, 4]	2,051 [8.7, 3]		742 [22.1, 4]	
				2,110 [28.0, 4]		2,056 [14.8, 3]		575 [15.2, 4]	
				2,490 [36.1, 4]					
				1,993 [58.9, 4]					

Note: Numbers in parentheses are N application levels (kg/ha).
Numbers in brackets are coefficient of variation and number of samples, respectively.
N application levels for each location do not vary by year.
L1 is Stillwater (site A), L2 is Haskell, L3 is Hennessey, L4 is Lahoma, L5 is Stillwater (site B), L6 is Perkins (site A), L7 is Perkins (site B), L8 is Stillwater (site C), and L9 is Tipton.
Weather data from each of these sites are available from Oklahoma Climatalogical Survey in Norman, OK, USA.

toward overestimating the slope and thus underestimating the N needed to reach the plateau.

Parameters in Eq. (4) are estimated with a linear mixed effects model (PROC MIXED in SAS). The null hypothesis of no random effects is tested using a likelihood ratio test. The parameters of the stochastic plateau model in Eq. (5) are estimated using SAS NLMIXED. Then, the estimates from Eqs. (4) and (5) are used to construct Eq. (6), which is then used to simulate expected net returns for each of the five nitrogen fertilization strategies considered.

4.1. Simulation of expected net returns

Net returns on 250 sample grids within each of 250 sample years are simulated using the following steps. First, sample values for the error components in Eq. (6) are simulated using a random number generator. Errors are assumed normally distributed with mean zero and estimated variances provided from the regression procedures used to estimate Eqs. (4) and (5). Intercepts, slopes, and expected value of optical reflectance information at the plateau are also provided from these regression procedures. In addition to the error components, values of ORI_{it}^{S} and ORI_{t}^{NRS} are simulated for each grid and year of the sample. Moreover, per ha custom application rates \$15.12, \$9.24, and \$12.37 were used for the pre-plant NH_3 uniform rate, the late-winter UAN uniform-rate system, and the late-winter UAN precise-rate system, respectively. A zero level of nitrogen is assumed when expected net returns from application are negative.

The process for calculating sample values of ORI from the NRS is

$$ORI_t^{\mathrm{NRS}} = ORI_t^{M} + v_t + u_t, \quad (17)$$

and the process for calculating sample values for the ORI on an individual grid and year is described by Eq. (5). Again, we note that since the NRS is a strip applied across a large area of the field, the plateau spatial variability is assumed to average out to zero given that a substantial number of readings are taken from it.

After sample values for the errors and the ORI are simulated for each grid and year, the formulas for the optimal levels of nitrogen (Eqs. (7), (9), (11), and (12)) for each of the alternative systems can be used to generate samples of optimal nitrogen rates for each grid in each year. Systems (3) and (4) assume measurements are without error and use the exact production function that is simulated, and therefore, the results represent upper bounds on what these systems could achieve in practice. If the expected return from applying nitrogen is below the application cost, then no nitrogen is applied. The yield response function defined in Eq. (6) is then used to calculate sample values for wheat yield for each system, grid, and year in the sample. Net returns are then calculated as the difference between wheat revenue and the cost of nitrogen and nitrogen application expenses for each grid in the year. The Monte Carlo integration is then completed by averaging net returns across the sample of years for each system. The differences in the average profits between the precision systems and the conventional extension service-recommended system of applying 90 kg per ha of nitrogen pre-plant provide an approximation for how much a conventional winter wheat producer would be willing to pay for a plant-based precision system. For each system, a long-run average price of \$0.11 per kg is used for the expected price of wheat grain and market prices of \$0.33 and \$0.55 per kg are used for NH_3 and UAN, respectively (Oklahoma Department of Agriculture).

4.2. Gains in efficiency

There are two reasons that topdress N might be used more efficiently than anhydrous applied pre-plant. First, denitrification or leaching of the nitrogen could occur. This is most likely to happen with sandy, high pH soils, and high rainfall. Second, the foliar applied UAN might be more efficiently absorbed by the plant than the soil applied anhydrous. Foliar applied UAN also has possible disadvantages in that it needs rainfall to make it available to the wheat plant and to prevent volatilization. The literature is mixed, with some experiments finding benefits from split or spring applications (e.g., Topal et al., 2003), while other experiments finding no benefit (e.g., Brown and Petrie, 2006). Woolfolk et al. (2002) and Subedi et al. (2007) provide brief reviews of other past experiments. Most of the soils dealt with here are fine-textured and of low pH, so leaching and denitrification should be limited. But many producers do use split applications, which suggests that there may be some benefit to spring applications. For this study, we assign a 20% gain in efficiency[2] to the marginal product of topdress nitrogen, such that the slope parameter β is effectively multiplied by 1.2. Note that such a linear relationship is a simplification and that topdress nitrogen may even be inferior for the initial units of applied N since many producers use split applications. The efficiency parameter is assigned to Eq. (6) and for the optimal levels of nitrogen in Eqs. (9), (11), and (12).

5. Results and discussion

Regression estimates of Eq. (4) are presented in Table 2. The null hypothesis of no random effects is rejected based on the likelihood ratio test. The slope parameter (b) is significant at the 0.05 level. The intercept parameter (a) is significant at the 0.10 level. Estimates of Eq. (5) are presented in Table 3. The marginal product of nitrogen ($1.2b\beta = (1.2 \times 452.1 \times 0.0265) = 14.39$) is considerably smaller than $\gamma/\tau = 0.6/0.0239 = 25.10$ assumed in the NFOA. Our estimate is that approximately 0.0695 kg of nitrogen should be applied to gain an additional kilogram of wheat, while Raun et al. (2002) assume that only 0.0398 kg is needed. The spatial variability is determined by the estimate of σ_ε^2 which is 0.5709. The relative variability of this estimate is within the range of spatial variability estimates found by Reed et al. (2008). Reed et al. considered a field

[2] Note that the optimal amount of nitrogen applied is not very sensitive to the price of nitrogen. For the perfect information system, we find that an average of 37 kg/ha of N is applied. If no efficiency adjustment is used with a price of N at \$.55/kg, then the net returns of the perfect information system will be reduced by \$4. Since the sensitivity to this assumption can be easily calculated, we do not include any formal sensitivity analysis with respect to the efficiency parameter.

Table 2
Regression of wheat yield response on the optical reflectance index (ORI)

Statistic	Symbol	Estimates[a]
Intercept	a	−351.2 (236.5)
Optical reflectance (ORI)	b	452.1 (28.2)
Year random effect	σ_ω^2	6,975 (1,736)
Error variance	σ_η^2	7,076 (358)

Note: The parameter estimates for Eq. (2) were estimated using PROC MIXED in SAS.
[a]Asymptotic standard errors are in parentheses.

Table 3
Stochastic linear plateau model of optical reflectance index as a function of nitrogen

Statistic	Symbol	Estimates[a]
Intercept	α	5.6882 (0.0640)
Level of nitrogen	β	0.0265 (0.1805)
Average plateau ORI	ORI_t^M	6.8879 (0.0599)
Nitrogen application at expected plateau	N_t^{NRS}	64.7410 (0.2193)
Variance of plateau yield	σ_v^2	0.6564 (0.1048)
Variance of year random effect	σ_u^2	0.8471 (0.0825)
Variance of error term	σ_ε^2	0.5709 (0.0295)

Note: The parameter estimates for Eq. (3) were estimated using NLMIXED procedure in SAS.
[a]Asymptotic standard errors are in parentheses.

selected because it had high spatial variability and most years the field had more spatial variability than that estimated here. Greater spatial variability would add to the value of the variable rate systems.

Expected yield, optimal levels of nitrogen, and expected profits for each system are reported in Table 4. As expected, the perfect information plant-based (variable topdress-sensed) precise-rate system based on real-time sensing and fertilization had the largest net expected return of approximately $271 per ha. The net return of this "perfect system" is approximately 6% greater than the (uniform pre-plant 90 kg per ha) extension service recommendation. The net expected return to the "perfect system" is $16 per ha relative to the uniform pre-plant 90 kg per ha system.

The late-winter topdress (with UAN) uniform-rate (uniform topdress-sensed) system, based on sensing of an NRS using perfect information, has an average net return approximately $9 per ha greater than that obtained from the uniform pre-plant 90 kg per ha (with NH_3) system.[3] The late-winter uniform topdress-sensed system uses 41% less nitrogen on average than the uniform pre-plant 90 system; however, the cost of nitrogen (as UAN) for the topdress system is only $0.49 per ha less than the cost of nitrogen (as NH_3) for pre-plant system. Moreover, the additional yield obtained with the uniform pre-plant 90 system relative to the uniform topdress-sensed system results from using a greater average uniform level of nitrogen (90 kg versus 53 kg). Using the average from a set of sensor readings taken from the farmer's field to approximate the uniform level of nitrogen needed to achieve the yield plateau is likely to result in some areas of the field receiving less nitrogen than actually needed, keeping some areas of the field from reaching the yield plateau.

A noteworthy comparison is the $16 per ha difference in net return between the perfect information (variable topdress-sensed) system and the system that uses the NFOA (variable topdress-sensed NFOA). The NFOA is unlikely to ever perform as well as the perfect information system described in this paper. However, the marginal product of nitrogen for the NFOA is too high, and reducing it to the actual size would presumably yield higher profits.

Sensitivity analyses with respect to changes in the price of wheat, the price of NH_3, and the price of UAN are reported in Table 5. Since most of the systems have yields close to the maximum, the differences in the systems vary little with respect to changes in price. The exceptions being the check (zero nitrogen fertilizer) system and the NFOA system, which both have lower yields. Where wheat price increases to $0.184 per kg, the value of the perfect information (variable topdress-sensed) system above that of the extension service recommendation (uniform pre-plant 90) is only an additional $1.85 per ha above what it was assuming $0.074 per kg.

As expected, the value of the perfect-information (variable topdress-sensed) system increases relative to the extension service-recommended (uniform pre-plant 90) system as the price of NH_3 increases relative to the price of UAN. When the price of NH_3 is increased to the point where it is equal to the price of UAN, the value of the variable rate precision (variable topdress-sensed) system increases to approximately $36 per ha over that of the extension service-recommended (uniform pre-plant 90) system. The opposite relationship exists when the price of UAN increases relative to the price of NH_3. If the price of UAN increases to $1.21 per kg, holding the price of NH_3 constant at $0.33 per kg, then the value of the extension service-recommended (uniform pre-plant 90) system is approximately $8 per ha more profitable than the perfect variable rate (variable topdress-sensed) system. In this situation, a typical producer would not be interested in adopting the plant-based precision system. If both NH_3 and UAN prices increase the

[3] Note that the extension recommendation is not the optimal pre-plant level. The optimal level can be computed following Tembo et al. (2008). The optimal pre-plant level is 65 kg/ha, which gives a yield of 2693 kg/ha and a net return of $260/ha.

Table 4
Average yield, nitrogen, and expected profits from alternative nitrogen management systems

Estimate	System				
	Zero[a]	Uniform pre-plant 90[b]	Uniform topdress sensed[c]	Variable topdress sensed[d]	Variable topdress sensed NFOA[e]
Average yield (kg/ha)	2,199	2,723	2,689	2,740	2,476
Average nitrogen (kg/ha)	0	90	53	37	18
Expected profit ($/ha)	242	256	265	271	255

Note: The prices used were $0.11/kg for wheat, $0.33/kg for NH_3, and $0.55/kg for UAN.
[a]A check system with no nitrogen added either pre-plant or late winter.
[b]A pre-plant NH_3 uniform rate of 90 kg/ha of nitrogen, a proxy for the state extension service recommendation.
[c]A late-winter topdress UAN uniform-rate system based on sensing of a nitrogen-rich strip (NRS) applied pre-plant to a narrow strip across the field (perfect knowledge).
[d]A late-winter topdress UAN precise-rate system based on real-time sensing (perfect knowledge).
[e]A late-winter topdress UAN precise-rate system based on real-time sensing and a nitrogen fertilizer optimization algorithm (NFOA) developed by Raun et al. (2002).

Table 5
Sensitivity values for independent relative changes in price of wheat, price of anhydrous ammonia (NH_3), and price of urea-ammonium nitrate (UAN) ($/ha)

Parameter	Price	System				
		Zero[a]	Uniform pre-plant 90[b]	Uniform topdress sensed[c]	Variable topdress sensed[d]	Variable topdress sensed NFOA[e]
Wheat price (p)	0.074	161	155	168	171	165
($/kg)	0.110	242	256	265	271	255
	0.147	323	356	363	372	348
	0.184	404	456	461	473	437
NH_3 price (r^P)	0.33	–	256	–	–	–
($/kg of N)	0.55	–	236	–	–	–
	0.73	–	220	–	–	–
	0.88	–	206	–	–	–
	1.10	–	186	–	–	–
UAN price (r^T)	0.55	–	–	265	271	255
($/kg of N)	0.77	–	–	257	263	251
	0.99	–	–	248	255	248
	1.21	–	–	243	248	246
p, r^P, r^T	0.22, 0.73, 1.21	484	520	532	549	516

[a]A check system with no nitrogen added either pre-plant or late winter.
[b]A pre-plant NH_3 uniform rate of 90 kg/ha of nitrogen, a proxy for the state extension service recommendation.
[c]A late-winter topdress UAN uniform-rate system based on sensing of a nitrogen-rich strip (NRS) applied pre-plant to a narrow strip across the field.
[d]A late-winter topdress UAN precise-rate system based on real-time sensing (perfect knowledge).
[e]A late-winter topdress UAN precise-rate system based on real-time sensing and a nitrogen fertilizer optimization algorithm developed by Raun et al. (2002).

same percentage to $0.73 and $1.21 per kg respectively, then the 90 kg per ha system has about $25 per ha less profit than all three systems that use sensing. But, applying no nitrogen is almost as profitable as using the sensing systems. In the presence of such large increases in nitrogen prices and $0.11 per kg wheat price, it is likely that farmers would recognize that it did not pay to apply nitrogen even if they did not use a sensing system.

The last line of Table 5 gives a scenario of high nitrogen and high wheat prices. The scenario is close to expected prices at planting in 2007 and uses the historical price ratio between UAN and NH_3. Actual UAN prices in 2007 were relatively higher. This scenario shows greater relative profit for the two perfect information systems. The variable rate perfect information system has a gain of nearly $29 per ha over the conventional practice of 90 kg per ha.

6. Summary and conclusions

Panel data covering nine years and seven locations in Oklahoma are used to estimate the wheat yield response to nitrogen conditional on optical reflectance information taken from growing wheat plants in the late winter. Only data from pre-plant applications were available and so the analysis is based on the assumption that there is a 20% gain in efficiency when using topdress applications. Under the assumption that the random processes are known perfectly, a maximum threshold value for the plant-based precision technology (variable topdress-sensed), over and above that of the conventional (uniform pre-plant 90) system, is found to be approximately $16 per ha. The field-level precision (uniform topdress-sensed) system is about $9 per ha more than the conventional system. With the 2007 prices for nitrogen and wheat, the advantage of the sensing systems approximately doubled.

The key reason that the potential advantage of the precision systems is so small is that pre-plant nitrogen can use anhydrous ammonia, which is much cheaper per unit of nitrogen. The value of the perfect information plant-sensing system found in this study would be greater if the price of NH_3 were to increase relative to UAN.

The findings of this study appear to explain why adoption has been slow. These findings also indicate that the optical sensing technology, including the NFOA in many cases, does not apply enough nitrogen fertilizer, and therefore could be improved (the formula has subsequently been changed to apply more nitrogen). Another disadvantage of the sensing system is that it is not calibrated to work when wheat is grazed. Also, there is a risk that fields could be too wet to apply any nitrogen in the spring.

While the findings of this study may be disappointing to some, this does not mean that there is no future to precision sensing for nitrogen. First, the study does show potential benefits from sensing. Even $9 per ha can become a large number if the technology can be used across a large area. The Raun et al. (2002) algorithm is continually being updated and improved. Less costly and less accurate systems such as a regional recommendation are a possibility. Tembo et al. (2008) estimate a higher marginal product of nitrogen than that estimated here, which would increase the value of sensing. There are potential disadvantages of applying excess nitrogen pre-plant that are not considered here. Research has occasionally shown yield losses from applying excess nitrogen. The technology may have more potential in a crop like sugar beets, where excess nitrogen promotes growth of sugar beet tops at the expense of root development. The excess nitrogen may increase acidity in the soil, which would cause producers to incur costs of applying lime. While most of the excess nitrogen is likely released to the atmosphere, there are potential externalities if the excess nitrogen leaches into groundwater or surface water. Also, some producers may not desire to use anhydrous ammonia and in that case the value of precision sensing would be much higher. While the potential returns may not be phenomenal and the precision sensing system analyzed may have net returns near breakeven levels, the research does show that returns to precision sensing could be positive if more accurate and/or less costly.

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