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Generalized Algorithm for Variable-Rate Nitrogen Application in Cereal Grains

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ABSTRACT

Many different mathematical algorithms have been developed and used in conjunction with commercial sensors for sensor-based nutrient management. Several of the N algorithms have led to the precise mid-season prediction of yields and calculation of sidedress N rates. The original Oklahoma State University (OSU) algorithm identified several limitations that were addressed in this study. Based on data analyses from more than 390 winter wheat (*Triticum aestivum* L.) and 200 corn (*Zea mays* L.) experiments and analyses of more than 100 N-rich strips, a generalized algorithm (for both corn and wheat) was developed to estimate the optimum N application rate based on spectral measurements. The generalized model adjusts the yield calibration curve for growth stages and better predicts corn and wheat yields. The coefficients of determination of the generalized model explained 5 to 6% less of the model error than the individual regressed data for both crops. Mean absolute error (MAE) was approximately 0.9 Mg/ha greater with the generalized model than with the individually regressed model. The larger MAE with the OSU generalized model was due to sensitivity to location of the inflection point; however, this sensitivity did not impact the calculated fertilizer rates. The generalized model reported here using normalized difference vegetation index sensor measurements collected midseason can be used to apply fertilizer N with changing growth stage for both corn and wheat.

The invention of analog-based, pulse-modulated, two-band, active lighting sensors (Beck and Vyse, 1994, 1995) and the equivalent digitally based sensor (Stone et al., 2003, 2005) have contributed to the potential use of these technologies for variable-rate application of N fertilizers. One of the more common reflectance indices used in agriculture is the normalized difference vegetation index (NDVI). The index is computed as (NIR – Red)/(NIR + Red), where NIR is the fraction of emitted near-infrared radiation returned from the sensed area (reflectance) and Red is the fraction of emitted red radiation returned from the sensed area (reflectance). Work by Filella and Penuelas (1994) and Liu et al. (2004a) noted that red edge reflectance can be indicative of plant chlorophyll content and biomass. Kanke et al. (2011) reported that NDVI better detected differences in plant growth, especially at early growth stages, than red edge reflectance. Spectral measurements of plants correlated with numerous physiological and morphological factors affecting growth and yield. Because of the difficulty in accounting for all confounding factors, models for computing N fertilizer rates are generally empirical and plant species specific and do not account for environmental factors, particularly rainfall, and their interactions with plant growth factors.

Biggs et al. (2002) proposed a reference strip, where fertilizer is applied at a sufficient rate such that crop yield reaches a response plateau, that would subsequently be used to manage N fertilization. He patented a concept to measure reflectance with an optical sensor of the strip and the adjacent field rate and calculated the N application rate based on the ratio of the two readings (Biggs et al., 2002). The sensors were mounted on a center pivot irrigation system and paired measurements were made on-the-go.

Researchers use linear or exponential models to describe the relationship between vegetative indices and plant yield. Linear relationships have been identified between yield and NDVI for corn (Diker et al., 2004), wheat (Nidumolu et al., 2008; Liu et al., 2004b), tomato (*Solanum lycopersicum* L.) (Bala et al., 2007), cotton lint (*Gossypium hirsutum* L.) (Plant et al., 2000), and barley (*Hordeum vulgare* L.) (Kancheva et al., 2007). Multiple linear regression was used for winter wheat (Salazar et al., 2006; Kumar et al., 1999). Exponential relationships were used for NDVI and yield in cotton lint (Plant et al., 2000), winter wheat (Enclona et al., 2004; Raun et al., 2005), spinach (*Spinacia oleracea* L.) (Jones et al., 2007), canola (*Brassica napus* L. var. *napus*) (Osborne, 2007), and corn (Raun et al., 2005). One model incorporated additional variables to account for other confounding factors such as the date of planting (Kumar et al., 1999). A comprehensive theory is needed to account for effects of the growth stage, rate of growth, date of sensing, and environment on crop growth and yield.

Raun et al. (2005) recognized that N algorithms should account for the independence of the crop response to additional N and potential maximum yield. As such, they must be measured individually. Because N is highly mobile (Khosla and Alley, 1999), the maximum potential crop yield is temporally and spatially (Girma et al., 2007) variable, and the amount N

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Abbreviations: MAE, mean absolute error; NDVI, normalized difference vegetation index; OSU, Oklahoma State University.

available from soil nitrification or denitrification varies greatly from year to year (Johnson and Raun, 2003). Furthermore, there is a strong agronomic basis for the argument that N algorithms must account for these factors by year and location. Any algorithm that combines the two without considering their independence will result in flawed recommendations (Raun et al., 2011).

Algorithms using other strategies, such as the sufficiency concept for recommending fertilizer N (Varvel et al., 2007), do not account for the temporal variability of these factors. An example of the sufficiency approach is work done by Varvel et al. (2007), which used normalized chlorophyll meter readings and relative or normalized yields to calculate N application rates. The use of a sufficiency index approach is appropriate for soil nutrients that are immobile, but models based on data averaged across years disregard the variability of yield responsiveness to N applied preplant and the yield response to unlimited N, both bound by the environment (year). As a result, the final N rate recommended is fixed to a sufficiency percentage determined from historical data and not tied to the yield level that would be achievable that year. Furthermore, the potential yield achievable is fundamental to calculating the total N demand for cereal crops in any crop year.

Lukina et al. (2001) proposed that the midseason N fertilizer required to maximize the grain yield for a specific season could be used to calculate the midseason N application rate. They proposed the following to predict the N application rate: $[(YP_{max} - YP_0)GN]/0.70$, where YP_{max} is the maximum potential yield, YP_0 is the potential yield with no additional fertilizer, GN is the predicted amount of total N in the grain, and 0.70 is the expected efficiency of the N fertilizer under ideal conditions. This method of determining in-season fertilizer need was shown to decrease large-area N rates while increasing wheat grain yields when each 1-m^2 area was sensed and fertilized independently. Later research by Raun et al. (2005) suggested that midseason N fertilizer rates be based on predicted yield potential and a response index. Their work showed that they could increase the N use efficiency by >15% in winter wheat, compared with conventional methods, at a 0.4-m^2 resolution.

Ferguson et al. (2002) suggested that improved recommendation algorithms may often need to be combined with methods such as remote sensing to detect the crop N status at early, critical growth stages followed by carefully timed, spatially adjusted supplemental fertilization to achieve optimum N use efficiency. Later work by Noh et al. (2005) confirmed that it was technically feasible to design a machinery-mounted multispectral imaging sensor to reliably and accurately detect crop N stress.

Zillmann et al. (2006) indicated that sensor-based measurements can be used efficiently for variable N application in cereal crops when N is the main growth-limiting factor. They further cautioned that the causes of variability must be adequately understood before sensor-based, variable-rate fertilization can be properly used to optimize N sidedressing in cereals.

Tubaña et al. (2008) found that using an algorithm that predicted the N responsiveness and yield potential (YP_0), with a modification for plant stand estimated using the coefficient of variation from sensor readings, resulted in net returns to N fertilizer that were higher when spatial variability was treated at <13.4-m^2 resolution (they also tested 0.84 and 26.8 m^2). Limited work has attempted to treat spatial variability in N at this small of a scale.

Ortiz-Monasterio and Raun (2007) showed that using a combination of an N-rich strip, together with the use of a GreenSeeker sensor and an algorithm to interpret the results from the sensor, allowed farmers to obtain significant savings in N use and thus farm profits. Farm income was increased by US$56/ha when averaged across all trials and years.

MATERIALS AND METHODS

Lukina et al. (2001) developed a methodology that accounted for the spatial variation in crop response to additional N fertilizer. With time, this was modified to address the temporal variability in N response (Mullen et al., 2003) and that led to a procedure to calculate N fertilizer application rates using optical sensor measurements of plant reflectance (Raun et al., 2005). In this algorithm, they delineated the following:

1. Measurements of light reflectance in one or more bands could be used to estimate a crop's potential yield during the growing season.
2. The ratio of paired reflectance measurements of an actively growing crop in an area of a field with standard fertility practices compared with similar measurements of the crop with nonlimiting N fertilizer in an immediately adjacent area could be used to determine the quantitative yield response with sufficient N fertilizer.
3. There is a maximum potential yield for any field or soil type within a field for any year, and this yield serves as a cap on yield that is independent of the amount of N fertilizer applied.
4. The fertilizer N application rate may be based on the mass of N removed in the grain at harvest. The N mass removal is a function of the difference in yields between a well-fertilized crop and the crop fertilized using the farmer's practice, multiplied by the percentage of N contained in the grain of the two, and divided by the efficiency at which N is taken up by the crop and processed into grain.

Extensive research, principally on wheat, established that the NDVI calculated using red and near-infrared reflectance was a good predictor of potential yield. This index was evaluated by Wanjura and Hatfield (1987) and is a good predictor of living plant biomass and grain yield, and is correlated to numerous plant stressors. The NDVI is calculated by

$$\mathrm{NDVI} = \frac{\rho_{\mathrm{NIR}} - \rho_{\mathrm{Red}}}{\rho_{\mathrm{NIR}} - \rho_{\mathrm{red}}} \qquad [1]$$

where ρ_{NIR} and ρ_{Red} are the reflectance values in the near infrared (780–880 nm) and red (650–670 nm) wavelengths, respectively. Although the original and generalized models use the spectral NDVI, other spectral indices and spectral bands may be used in these algorithms (Shaver et al., 2011). A simple exponential growth model is commonly used to predict the grain yield potential (YP):

$$\mathrm{YP} = a\exp(b\mathrm{NDVI}) \qquad [2]$$

where coefficients a and b are determined empirically. Raun et al. (2005) showed that the regression coefficient for the model could be improved by dividing the NDVI by the number of days with active growth, particularly for winter wheat. They defined the index created from these variables as INSEY, or in-season estimate of yield, where INSEY = NDVI/number of days from planting to sensing during which growth was possible. Growing-days equations are a function of the plant species. For example, days when winter wheat grows actively or when growth is possible occur when the daily average temperature is ≥4°C (Porter and Gawith, 1999). From data collected between 1998 and 2003 at 30 locations, Raun et al. (2005) modified the exponential growth model as

$$\mathrm{YP} = 0.359 \exp(324.4\ \mathrm{INSEY}) \quad [3]$$

To be able to make correct N recommendations, establishing the plant response to applied N is important. Mullen et al. (2003) and Raun et al. (2005) observed that paired optical measurements of the crop with and without sufficient N could be used to estimate the increase in crop yield with additional N, which they termed the sensor response index, RI_{NDVI}:

$$RI_{NDVI} = \frac{NR_{NDVI}}{FP_{NDVI}} \quad [4]$$

where NR_{NDVI} is the average NDVI value taken from a strip where N is not a limiting factor and FP_{NDVI} is the average NDVI value measured from an adjacent strip with fertilizer applied at the field or farmer practice rate.

Their research demonstrated that NDVI calculated from these measurements could be used to predict yield with and without additional N. The potential yield with additional N, YP_N, was calculated as the product of RI_{NDVI} and the yield potential without additional N:

$$YP_N = RI_{NDVI} YP_0 \quad [5]$$

where YP_0 is calculated with Eq. [2] and the NDVI value of the farmer practice rate of the paired comparison is used to calculate the RI_{NDVI}. They noted, however, that YP_N could be calculated as

$$YP_N = RI_{NDVI}\, a \exp(b\mathrm{NDVI}) \quad [6]$$

During the process of developing the theory to determine the N application rate, they elected to use Eq. [3] with an adjustment equation to compensate for underestimating the harvest yield.

As implemented, YP_N was calculated as

$$YP_N = \min(RI_{NDVI} YP_0, YP_{max}) \quad [7]$$

Eleven years of testing the first N application algorithm (Lukina et al., 2001) identified the following issues and limitations:

1. There are discontinuities in the yield model.
2. The maximum potential yield is not incorporated into a continuous function.
3. Boundary conditions are not included, specifically:
    a. The yield potential, Eq. [2], does not predict zero yield on bare soil nor is it bounded at FP_{NDVI} = 1. At FP_{NDVI} = 1, the potential yield must equal the highest potential yield within a field. Consequently, an additional measurement was required to estimate the maximum yield potential.
    b. Equation [2] calculated the average potential yield of a number of fields across several years, not the yield in a particular field in a particular year.
4. Examination of field data collected during a 10-yr period indicated that the response index varied continuously across the entire range of the vegetative index (Raun et al., 2008).
5. Bare soil was arbitrarily defined as NDVI = 0.25.
6. Crop growth stage and varying rates of biomass at each growth stage are not fully accounted for.
7. A procedure is needed to incorporate the maximum potential yield for a specific field, a location within a field, and year, as well as growth stage, in the yield model.
8. Parameters need to be generated for each crop species.

To address these and related issues of existing models to predict N application rates, a generalized algorithm was created to predict the N application rate based on spectral measurements. The model was composed so as to require that boundary conditions be met without requiring maximum or minimum constraints and that the model be continuous across the range of application. The model was fit based on 7 yr of data acquired since the introduction of the original model (Lukina et al., 2001).

MODEL DEVELOPMENT

An examination of the data relating NDVI to measured yield accumulated during 11 yr at numerous locations demonstrated the need to revise the theory for predicting potential yield and response to additional N.

Generalized Theory for Topdress Nitrogen Application

Spectral and Agronomic Considerations

Researchers have known for some time that certain spectral wavelengths are associated with biological processes within a crop. In particular, the amount of energy in the red band that is absorbed by a plant is a function of the plant's photosynthetic potential. There is a finite amount of energy that can be absorbed by the plant, which in all cases is less than the amount of energy reaching the plant. The ratio of the incident and reflected light is the reflectance. The value of peak reflectance in the red band (ρ_{RED}) is typically 5 to 10%.

A second critical band is the near infrared, which measures a plant's ability to reflect unwanted light energy (heat) in the infrared region (wavelength $\lambda \geq 780$ nm). Reflectance in the near-infrared region (ρ_{NIR}) varies among species and is a function of the health of the plant. Typical reflectance in the near-infrared region is 50%.

Reflectance measurements of these bands (or other bands of interest) are generally incorporated into a spectral vegetative index. These indices serve several purposes. Of particular interest, they can increase the magnitude and sensitivity of the measurements and in the case of normalizing indices, remove or minimize the effect of other variables on the index value. These indices have been correlated with a number of plant responses including plant biomass and yield. Numerous spectral indices have been created, but for the purpose of creating an algorithm for N fertilizer application, the NDVI (Eq. [1]) will be used

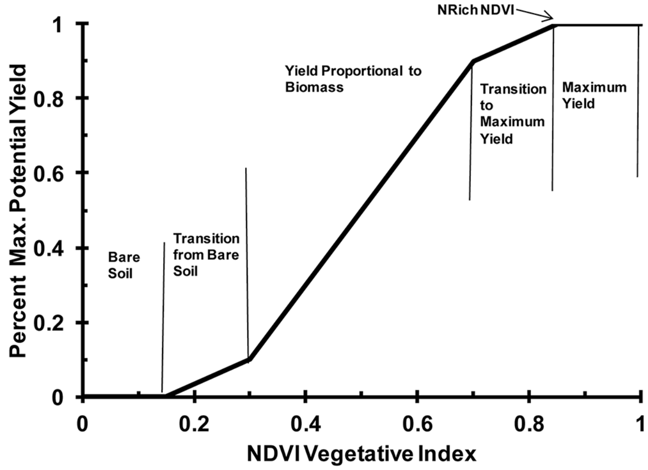

Fig. 1. Regions of model relating plant yield to the vegetative index inferred from the general knowledge of the relationships of plant biomass, normalized difference vegetation index (NDVI), and crop yield.

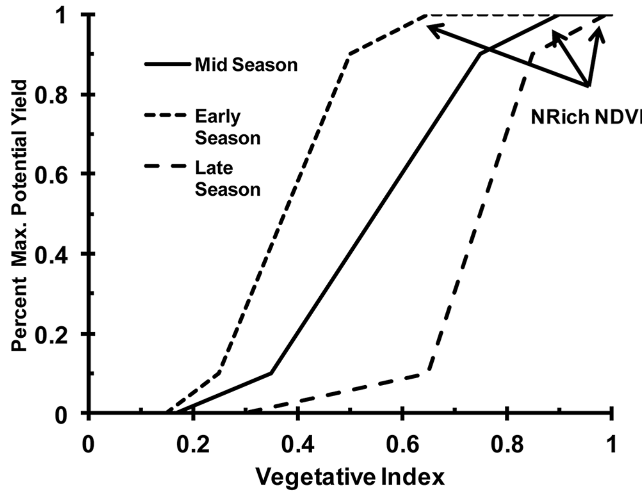

Fig. 2. Regions of model relating plant yield to the vegetative index inferred from the general knowledge of the relationships of plant biomass, plant growth stage, normalized difference vegetation index (NDVI), and crop yield.

to infer the shape of the yield prediction curve. Extensive research, principally on wheat, established that the NDVI calculated using red and near-infrared reflectance was a good predictor of potential yield. Kanke et al. (2011) showed that an alternative red-edge index (computed using two different methods) behaved very similarly to the NDVI in winter wheat.

The following inferences can be made on the shape of the curve created by the NDVI yield model based on knowledge of the NDVI and the general relationship of biomass and N nutrient management:

1. A yield model with NDVI as the independent variable will have no potential yield for any value of NDVI less than the bare-soil NDVI (Fig. 1).
2. There is a central region where potential yield increases rapidly with increasing levels of biomass (Fig. 1). Because biomass is roughly proportional to the NDVI, crop yield is also roughly proportional to the NDVI. The bulk of the variation in yield and biomass occurs in this region.
3. There is a transition region between bare soil and the central region where there is a limited increase in yield with increasing spectral index (Fig. 1). In this region, crop stands are poor or growth is retarded due to other agronomic factors.
4. The yield curve reaches a plateau, which is the maximum potential yield (Fig. 1). In this region there is limited or no response of yield to changes in spectral measurements. In this region, the photosynthetic potential is great enough to effectively use all available incoming light. There is sufficient N to produce the maximum potential yield, and crop growth is limited by other agronomic factors including the genetic yield potential of the cultivar.
5. There is a second transition region where potential yield changes to maximum potential yield. In this region, response to additional N fertilizer is limited (Fig. 1).
6. The point where the yield/NDVI curve reaches the maximum potential yield is the value of NDVI measured from the N-rich reference strip (Fig. 1). This would be the maximum yield under fully sufficient N or the reference strip where N is not limiting.
7. Although the general shape of potential yield curves are defined in Fig. 1, the model must account for the growth stage. Early in the growing season, differences in crop growth are obvious, but even plants with sufficient N have not closed the crop canopy. The NDVI values of these plants with nonlimiting N will be much less than NDVI = 1. Because the crop yield reaches a plateau at NDVI < 1, the curve relating the spectral index to the potential yield must shift to lower values of the vegetative indices, i.e., the curve must shift to the left (Fig. 2).
8. Similarly, at later growth stages the crop fully covers the canopy and even poor and irregular plant stands have achieved substantial growth. Values of the NDVI from the N-rich reference strip should then approach 1. The maximum yield plateau may be a point. The potential yield curve must be shifted to the right to account for higher spectral values (Fig. 2).

Nitrogen Rate Prediction Model

A parameterized symmetric sigmoid model with zero intercept satisfies these boundary condition requirements (Fig. 3). The model relating NDVI to yield is

$$\mathrm{YP} = \frac{\mathrm{YP}_{\max}}{1+\exp\left[-\left(\mathrm{FP}_{\mathrm{NDVI}}-\mathrm{Inf}\right)/K\right]} \qquad [8]$$

where YP_{max} is the maximum potential yield of the crop within a field or area within a field, Inf is the location of the inflection point of the model, which is the point where the predicted yield is one-half the maximum yield, and K controls the curvature of the sigmoid model, with the slope of the sigmoid at the inflection point decreasing with increasing values of K. In addition, because bare soil yields NDVI values generally appreciably greater than zero, the minimum value of the NDVI for calculation of the N rate must account for bare soil.

The sigmoid curve (Fig. 3) generated from Eq. [7] closely conforms to the agronomic requirements depicted in Fig. 1.

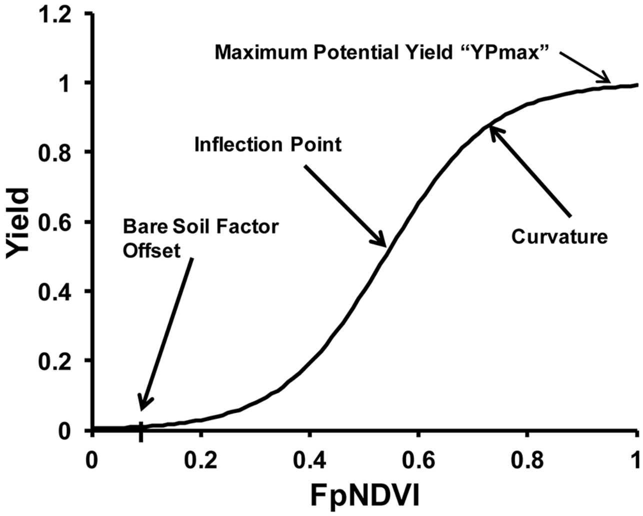

Fig. 3. Sigmoid yield model with critical parameters. The bare soil factor offset is 0.1; FP_{NDVI} is the average value of the normalized difference vegetation index (NDVI) measured from an adjacent strip with fertilizer applied at the field or farmer practice rate.

The mathematical model does not allow the curve to shift to reflect the growth stage of the crop. This is because the equation coefficients are constants; however, these coefficients can be converted to parameters that are continuous equations and functions of the independent variable, NDVI.

Model Validation and Parameter Regression

Model Validity

To empirically verify the validity of the sigmoid as a yield model, the model was evaluated against 390 site-year-date wheat experiments and 92 site-year-date corn experiments. These experiments were conducted from 1998 to 2008. All trials had in common three or more N fertilizer rates plus a zero-rate check. Some of the long-term experiments included in this study (Raun et al., 2001) evaluated other factors such as P, K, and S, but these treatments were not included in the analysis. All were optically sensed by either a GreenSeeker optical sensor (NTech Industries, Ukiah, CA) or the natural lighting sensor described in Lukina et al. (2001). Both sensors were calibrated against a $BaSO_4$ white plate. Those experiments that were optically sensed only once were dropped from the analysis because change in the sigmoid curve shape as a function of time was important for model testing.

The remaining experiment site-year data were nonlinearly regressed to a zero-intercept sigmoid model using the curve-fitting program TableCurve (Systat Software, San Jose, CA), as were the single-sensing site-years, which were included in the final validation of the model. Data sets that could not be regressed by TableCurve were removed. These data sets contained data that had no trend. This left 37 corn site-year-dates and 86 wheat site-year-dates to build the parameterized model. The TableCurve nonlinear regression was able to fit the zero-intercept sigmoid model, with all boundary conditions satisfied, for the remaining 123 site-year-date data sets for corn and wheat (site examples in Fig. 4a and 4b). Approximately 60% of the 390 site-year-date experiments were incomplete, and when regressed using TableCurve with the zero-intercept sigmoid model, all of the boundary conditions could not be satisfied (Fig. 4c). The example in Fig. 4c shows no clear maximum, thus no sigmoidal plateau could be defined within the NDVI boundary (0.0–1.0 NDVI). Other data sets like this, where maximum yields were not discernable, could not be used for developing the parameterized model. The balance of the data sets did not demonstrate any clear trends and usable sigmoid models could not be fitted to these site-years (Fig. 4d). These distributions were observed for corn (not shown) and wheat (shown). Once the parameterized model was created, however, the model was validated with all data sets, including those that did not have defined boundaries and data sets where there was no clear relationship between yield and the NDVI.

Defining Maximum Yield

The original theory allowed the producer, crop advisor, fertilizer dealer, agronomist, or other specialist to define the maximum expected yield for a specific year and field. Farmers, with long-term yield data coupled with their intuition, have demonstrated that they can make reasonable yield estimates within a field. An optical sensor was used to measure the NDVI along a non-N-limiting strip through a field. The maximum yield was predicted by Eq. [2]. Exponential models relating the NDVI to yield (e.g., Eq. [2]) can generally provide reasonable estimates of the expected maximum yield. These models are based on composited data for many site-years, however, and must be adjusted for the field and growing conditions (Raun et al., 2005).

Crop growth models can be useful in predicting maximum yield. Yield estimates can be made or adjusted based on fertilizer costs and the market value of the grain. In the absence of field-level yield data, county grain yield weighted by soil class and general weather conditions provides a basis on which to estimate maximum yield. Unfortunately, with existing tools, predicting maximum yield remains challenging.

Model Parameter Definition

The optical sensor measures bare soil, which typically has a bare-soil NDVI (BS_{NDVI}) value greater than zero, normally ranging between 0.12 and 0.25. Consequently, measurements with living biomass have NDVI values ranging from $BS_{NDVI} < NDVI \leq 1.0$. Because optical sensor measurements include bare soil, the model must satisfy two end conditions: (i) the model yield prediction must asymptotically approach the bare soil NDVI (the transition zone in the conceptual model), and (ii) the yield must asymptotically approach the maximum yield when NDVI = 1. The latter end condition can be derived from the mathematical definition of the NDVI because much of the incident red light is theoretically absorbed by the plant for photosynthesis.

To define the inflection parameter, Inf, as a function of the NDVI the nonlinear regressions of each data set (determined by TableCurve) were graphed against the maximum value of the NDVI in the experiment (Fig. 5). This value should be the value of the crop when N is not a limiting factor, NR_{NDVI}. The maximum NDVI value was assumed to be non-N-limiting and, in all cases, came from plots with the highest N application rates.

Inflection Point

Linear models were fitted to data from the 127 experiments with complete data sets (Fig. 5a). Before analyzing the data, we hypothesized that parameter values equaled zero on bare soil. This was clearly the case for the Inf in corn but not for wheat (Fig. 5b).

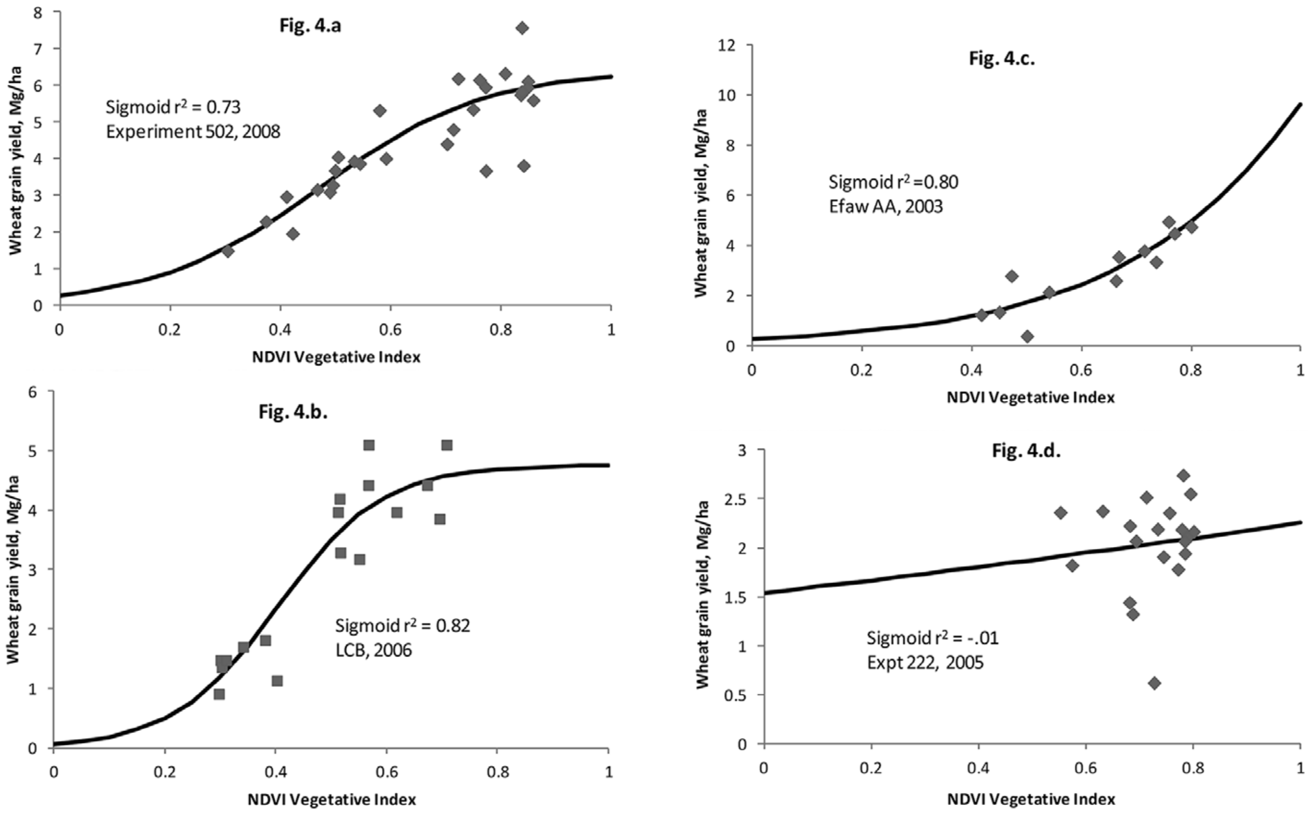

Fig. 4. (a,b) Typical sigmoid model nonlinear regressed yield and normalized difference vegetation index (NDVI) sensor data, with sufficient data to define the maximum potential yield and region of high response to change in the NDVI; (c) insufficient data to establish maximum potential within the genetic yield potential for hard red winter wheat under dryland conditions; and (d) the limited number of experiments where data were insufficient or so scattered to prevent fitting a sigmoid curve with certainty. Similar distributions were observed for corn.

The slope of the curve for wheat (Fig. 5b) was varied, and only a slight decrease in r^2 (data not reported) was observed with positive values of the bare-soil NDVI ($0.03 \leq BS_{NDVI} \leq 0.04$). Considering the number of site-year-date data sets incorporated into this analysis, the r^2 and MAE (Table 1) values are reasonable.

Curvature

The parameter K enables the boundary conditions of the sigmoid model to remain satisfied as the inflection point is shifted along the abscissa. To do this, the radius of curvature (or the tangent of the curve at the inflection point) must change.
In this case, where the abscissa is the NDVI, the inflection point values range from 0 to 1. Because the biomass increases throughout the growing season, the value of the NDVI for any area within a field increases with crop growth. This implies that although the biomass and NDVI change, the yield model must continue to satisfy the boundary conditions. This requirement can only be satisfied if K approaches zero at NDVI = 0 and NDVI = 1, i.e., the slope of the model approaches infinity at the inflection point. At NDVI = 0.5, K equals a finite value where all boundary conditions are satisfied. The same results were predicted qualitatively in Fig. 2.

A close examination of the curvature data in Fig. 6 and Table 1 indicates that:

1. Corn and wheat K responded the same to changes in the NR_{NDVI}.
2. Bare soil was not clearly a factor affecting K; however,

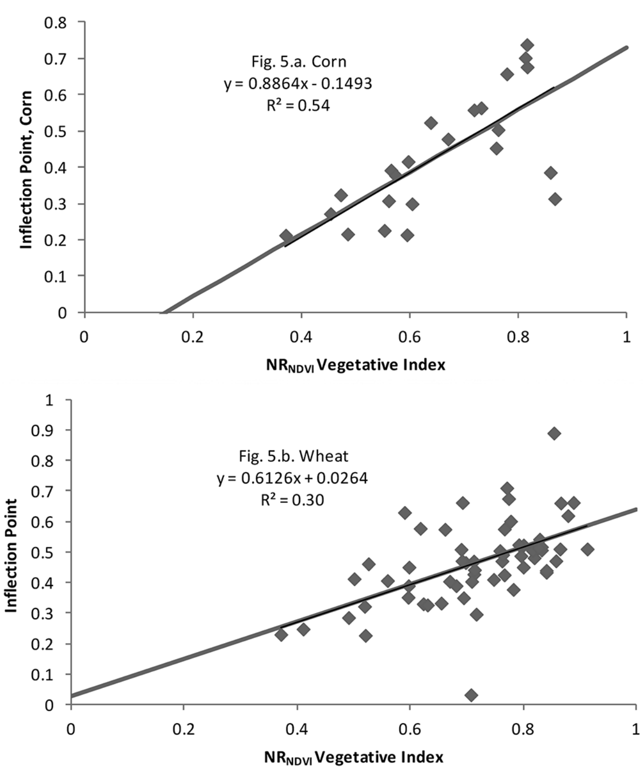

Fig. 5. Linear regression to determine the relationship of (a) corn and (b) wheat inflection points to the average value of the normalized difference vegetation index (NDVI) taken from a strip where N is not a limiting factor (NR_{NDVI}). The corn inflection point is highly sensitive to bare soil (NDVI = 0.17), while the wheat inflection point curve appears insensitive to bare soil.

Table 1. Comparison of generalized yield model using the normalized difference vegetation index (NDVI) fit to all corn and all wheat data with a sigmoid regression model fit to individual data sets.

Crop	Yield model	Parametric model	Bare- soil NDVI	Estimate	Sigmoid regression r^2†	Generalized model r^2‡	Sigmoid regression MAE§	Generalized model MAE¶	MAE difference
							Mg/ha		%
Corn	corn only	Inf = 0.738NDVI – 0.0480	0.068	mean	0.421	0.374	0.607	0.710	16.9
		K = 0.155NDVI – 0.0101		median	0.494	0.379	0.532	0.570	7.1
	corn and wheat	Inf = 0.773NDVI – 0.0479	0.065	mean	0.421	0.374	0.607	0.795	30.9
		K = 0.168NDVI – 0.0104		median	0.494	0.381	0.532	0.722	35.7
Wheat	wheat only	Inf = 0.808NDVI – 0.0477	0.059	mean	0.592	0.528	0.422	0.465	10.2
		K = 0.1923NDVI – 0.0113		median	0.593	0.529	0.381	0.405	6.4
	wheat and corn	Inf = 0.773NDVI – 0.0479	0.065	mean	0.592	0.525	0.422	0.505	19.7
		K = 0.168NDVI – 0.0104		median	0.593	0.537	0.381	0.447	17.5

† TableCurve sigmoid regression model r^2.

‡ Oklahoma State University (OSU) generalized model r^2.

§ TableCurve regression model mean absolute error.

¶ OSU generalized model mean absolute error.

r^2 was insensitive to the intercept value (Table 1).

3. Unexpectedly, both data sets exhibited a void in the data, and both corn and wheat data located above the void appeared to fall on the same curve (Fig. 6).

We contend that a single linear curve best represents the data. Values of K above the void were associated with NDVI–yield data sets with a limited range of NDVI or data sets with nearly constant values for yield for lower NDVI values. There were a few NDVI–yield data sets with high values of K. These outliers were responsible for the low r^2 values and MAE values for the parametric equation predicting K.

Model Optimization and Validation

Model optimization and comparison of the OSU yield model to the TableCurve sigmoid regression models were performed. To validate the hypothesis that a single model could be created to predict the yield of two morphologically different cereal grains (corn and wheat), the generalized model was fit to all crop-site-year-date data sets. This included data sets that were apparently incomplete because they did not depict the entire curve. Each data set was nonlinearly regressed with a zero-intercept sigmoid model and the maximum yield and r^2 recorded. The MAE was calculated. Equations to calculate the parameters using the generalized algorithm were incorporated into each data set and linked to a table of values for the constants of these equations. The parametric equations were formulated so that the BS_{NDVI} defined the intercept of the NDVI axis. The maximum yield value of the data set was used to set the maximum yield in the TableCurve regression. Data sets where TableCurve produced maximum yield values clearly outside of the ranges biologically possible for corn and wheat were excluded. Because constants in the equations were interlinked, they could be adjusted to optimize their values. Mean and median values of the MAE and r^2 were optimized for all data sets by adjusting the constants for the two parametric equations and BS_{NDVI}. The parametric equations were optimized for corn and wheat data individually and optimized as a single set of equations for all data.

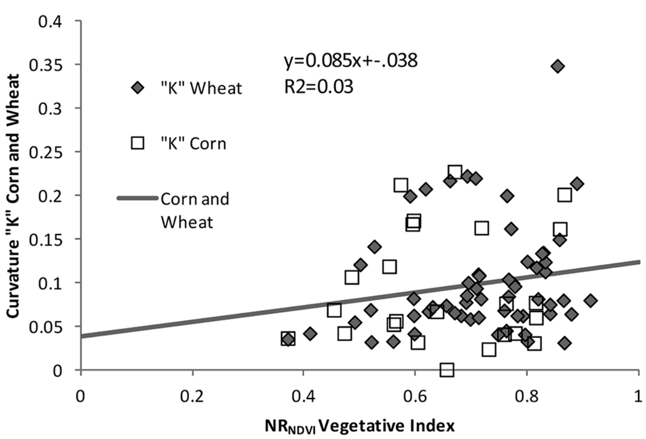

Fig. 6. Relationship between curvature K and the average value of the normalized difference vegetation index (NDVI) taken from a strip where N is not a limiting factor (NR_{NDVI}) for corn and wheat. Both species exhibited some data with high K values.

As expected, r^2 values for the generalized model were lower than for the individually regressed data. However, r^2 of the generalized model explained only 5 to 6% less of the model error than the individually regressed data for both crops. The MAE was approximately 0.2 Mg/ha greater with the generalized model than with the individually regressed model. The larger MAE with the generalized model was a result of sensitivity to the location of the inflection point, as shown below. This sensitivity had little effect on fertilizer rates.

It had been assumed that BS_{NDVI} could be included directly to generate the parametric equation. Optimization showed, however, that bare soil had less of an effect than expected (Table 1). The implication is that considerable error can occur in the transition region between bare soil and the crop, but because neither corn nor wheat should be fertilized in this region, any error in fertilizer rate will be small.

Several important observations can be gleaned from Fig. 7 and 8:

1. The generalized models were better predictors of corn yield and conformed closely to the regression model.
2. The generalized model adjusts the yield calibration curves for growth stage as days from planting increase. It does so by compensating for plant growth (accumulated living biomass). The fundamental assumption of N-rich strips or N reference strips is that the rate of plant growth is proportional to the available N in the root zone. Once plants begin growing vigorously, the differentiation in growth rate (as indicated by cumulative biomass) is easily observed and measured

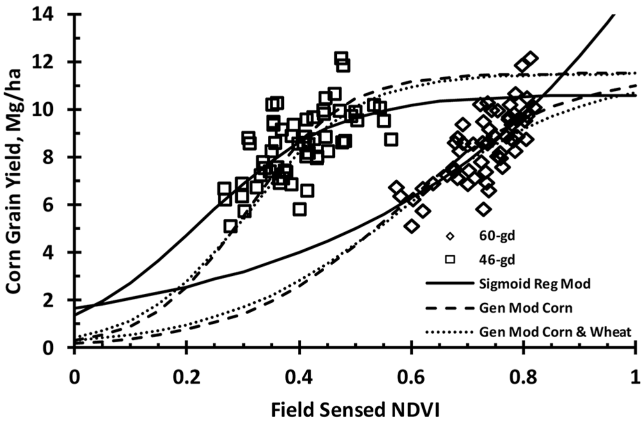

Fig. 7. Predicted corn yield sensed 46 and 60 d after planting based on field-sensed normalized difference vegetation index (NDVI) using sigmoid regression, the generalized model for wheat, and the generalized model for corn and wheat.

with a sensor. This differentiation generally begins in wheat at growth stage Feekes 4 and in corn by the five-leaf stage. The differentiation caused by variations in the available N continues until flowering. The sensor measurement of the non-N-limiting N-rich or reference N strip increases with growth stage, but in all cases the value (of NDVI) is equivalent to the potential grain yield when N is nonlimiting. In other words, the sensor reading is an analog for both accumulated biomass and potential grain yield at any time after the plant reaches the minimum growth stage.

3. The range in NDVI for the earliest sampling dates was narrow, which was reflected in the poor r^2 values for all models. These sampling dates occurred before the date when the crops fully displayed available N. Nevertheless, the generalized model could be used to apply N differentially based on optical sensor measurements.

4. The magnitude of BS_{NDVI} did not directly affect the parametric equations, which were optimized for a BS_{NDVI} value of approximately 0.06. This is a consequence of the zero-intercept symmetric sigmoid function, which does not necessarily approach zero yield at zero NDVI and whose fit is not optimized at BS_{NDVI}; however, BS_{NDVI} should be incorporated into the algorithm to halt fertilizer application on bare soil.

Calculating Nitrogen Application Rate

The N fertilizer application rate was calculated using an approach similar to Lukina et al. (2001). Grain yield with sufficient N was calculated as

$$YP_N = \frac{YP_{max}}{1+\exp\left[\left(RI_{NDVI}FP_{NDVI}-Inf\right)/K\right]} \qquad [9]$$

This equation incorporates the expected increase in yield with additional fertilizer, incorporating RI_{NDVI} to calculate the value of NDVI with sufficient N at a specific location. The value of RI_{NDVI} is calculated from the N-rich calibration strip using Eq. [3].

Nitrogen Topdress Rate Calculation

The N application rate (N_{rate}) can be calculated using the same equation used in the functional algorithm (Lukina et al., 2001):

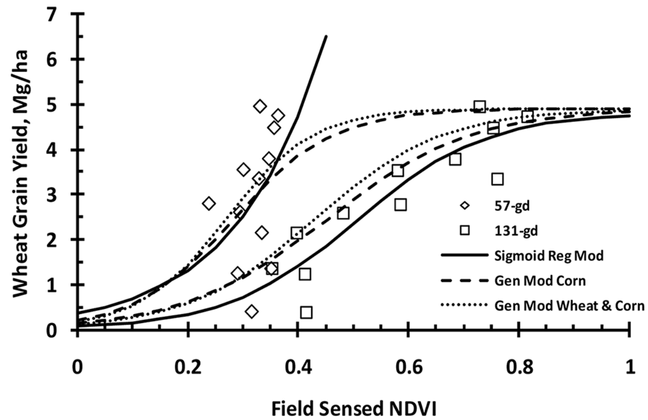

Fig. 8. Predicted wheat yield sensed 57 and 131 d after planting based on field-sensed normalized difference vegetation index (NDVI) using sigmoid regression, the generalized model for wheat, and the generalized model for wheat and corn.

$$N_{rate} = \frac{\left(YP_N - YP_0\right) - \%N}{NUE} \qquad [10]$$

where %N is the percentage of N contained in the grain or forage and NUE is the N use efficiency.

The N rate increases with the NDVI to a point where the potential yield is achieved. Beyond that, any increase in the NDVI will lead to a decline in the N rate applied (Fig. 9). If a high NR_{NDVI} is used in calculating the N rate, the dispersion of the curve will be wider than with a lower NDVI, but the N rate required to attain the potential yield will stay the same. Although the trend remains the same, the curve with which the N rate was calculated using 0.4 NR_{NDVI} shifts left, while the curve using 0.8 NR_{NDVI} shifts to the right (Fig. 9). The parameter K responds to changes in NR_{NDVI} and maintains the boundary conditions of the sigmoid model as the inflection point is shifted along the abscissa.

Algorithm to Calculate Nitrogen Fertilizer Application Rates

The following procedure implements the generalized algorithm for calculating application rates for variably applied N fertilizer:

1. Estimate the maximum yield for the field under current growing conditions. We have developed a method to

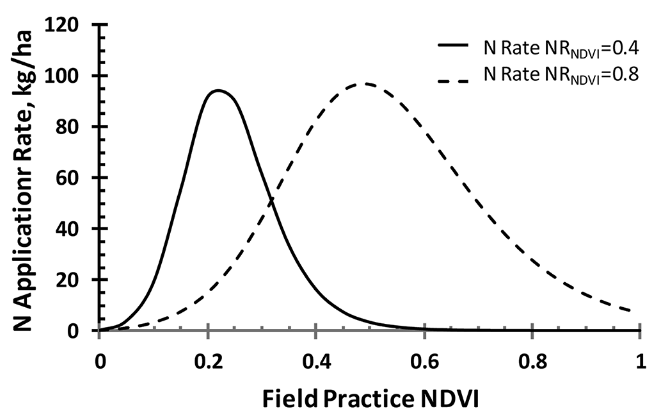

Fig. 9. The effect of the normalized difference vegetation index (NDVI) taken from a strip where N is not a limiting factor (NR_{NDVI}) on the N application rate.

Table 2. Comparison of the generalized yield model fitted to all corn and all wheat data with a sigmoid regression model fitted to individual data sets.

Crop	Experiment	Plant growth present	Yield model	Sigmoid regression r^2†	Generalized model r^2‡	Sigmoid regression MAE§	Generalized model MAE¶	MAE difference from regression model
		d				——— Mg/ha ———		%
Corn	LCB 2006, 99-d maturity	46	corn only	0.365	0.160	0.876	1.081	16.9
			corn and wheat	0.365	0.172	0.876	1.216	30.9
		60	corn only	0.533	0.347	0.779	0.810	4.0
			corn and wheat	0.533	0.362	0.779	0.947	21.6
Wheat	LCB 2006	57	wheat only	0.141	0.107	1.114	1.129	1.3
			wheat and corn	0.141	0.101	1.114	1.089	0.0
		131	wheat only	0.821	0.816	0.502	0.517	3.0
			wheat and corn	0.821	0.809	0.502	0.630	25.4

† TableCurve sigmoid regression model r^2.

‡ Oklahoma State University (OSU) generalized model r^2.

§ TableCurve regression model mean absolute error.

¶ OSU generalized model mean absolute error.

estimate the potential yield for winter wheat. Models exist with varying degrees of complexity that are accurate in predicting yield. Our experience is that farmers with good yield records can estimate grain yields midway through the growing season with a reasonable degree of accuracy and precision.

2. Measure the NDVI of paired areas in an N-rich strip and adjacent farmer practice strip**.** These areas should be located in a region of the strip where the topography and soils are uniform. Measuring in areas where the effect of sufficient N is obvious will improve the precision of subsequent calculations. Four to five paired measurements will improve the precision of the calculations. The most responsive of these measurements should be used for further calculations.
3. Define the potential yield curve**.** Determine which equations (corn, wheat, or corn and wheat combined, Table 2) will be used to calculate the yield curve parameters curvature (K) and the inflection point (Inf). Use the N-rich (reference) NDVI from the N-rich strip. Enter the maximum expected yield within the field, YP_{max}, into the yield model to create the upper boundary for the model.
4. Calculate the response indices from the paired N-rich strip measurements**.** Use Eq. [4] to calculate RI_{NDVI} (one to five values).
5. Calculate the values of the coefficients for the parameters Inf and K. Use the appropriate equations in Table 1. Use the value for NDVI (RI_{NDVI}) from the N-rich reference strip.
6. Scan the area to be treated.
7. Calculate the expected yield curve without additional N using Eq. [8].
8. Calculate the yield with additional fertilizer using Eq. [9], which incorporates the response index.
9. Calculate the N fertilizer application rate. Values for the percentage of N in grain, forage, fruit, and other crops are well known. Nitrogen use efficiency is typically between 50 and 60% for topdress or sidedress fertilizer applications. Equation [10] provides a method for calculating the application rate. Standard equations should be used to adjust the rates for the specific forms of fertilizer.

CONCLUSIONS

The improved models for predicting the yield potential and the response index overcome the limitations of the Lukina et al. (2001) algorithm. The equations and procedures presented here are based on data from more than 200 corn and wheat experiments and analyses of more than 100 N-rich strips. Reflectance measurements in the red and near-infrared bands are incorporated into the spectral index (NDVI), which correlates with plant biomass and yield. Yield models with NDVI have zero potential yield for values less than the BS_{NDVI}. It was established that yield increased with biomass and that there is a transition region between bare soil and the central region, where there were limited increases in yield with increasing NDVI. The response to additional N was limited when the potential yield approached the maximum potential yield. The OSU generalized model resulted in equations for the calculation of the N rate by introducing new parameters: the inflection point (Inf) and the curvature (K). The inflection point, ranging from 0 to 1, equaled zero on bare soil in corn but not in wheat. The slope of the curve for wheat had a slight decrease in r^2 with positive BS_{NDVI} values ($0.03 \le BS_{NDVI} \le 0.04$). The curvature enabled the boundary of the sigmoid model to be satisfied when K approaches zero at NDVI = 0 and NDVI = 1 as the inflection point shifts along the abscissa. Corn and wheat K responded to changes in NR_{NDVI} and bare soil was not a factor, although the BS_{NDVI} should still be incorporated into the algorithm to stop fertilizer application on bare soil. Based on the equations in the model, a single model could be created to predict the yield of two morphologically different cereals (corn and wheat). The generalized model was able to adjust for growth stage and better predict corn and wheat yields. The range in NDVI values recorded from very early growth stages were narrow and were reflected in poor r^2 values. These coefficients were lower for the generalized model than the individual regressed data; however, the r^2 of the generalized model explained only 5 to 6% less of the model error than the individual regressed data for both crops. The MAE was approximately 0.9 Mg/ha and was greater with the generalized model than with an individually regressed model. The larger MAE with the OSU generalized model was due to sensitivity to location of the inflection point; this sensitivity, however, had little effect on the fertilizer rates. The error could also occur in transition regions, but neither wheat nor corn benefit greatly from fertilizer applied in these regions. The generalized model reported here using NDVI sensor measurements can be used to apply fertilizer N with changing stages of growth for both corn and wheat.

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